A Rare Accident

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Notwithstanding the great efforts made during the past decade to explore the domain of modal systems between S4 and S5, it may not be very unusual that new systems are continually discovered. Neither may it be extraordinary that one such new system is discovered independently by two different authors. It seems however, to be a rare accident, indeed, if two authors independently discover one and the same system by means of different proper axioms and each decides to give his system one and the same name.

In [3], I had considered among others a new modal system which results from appending to S4 the following substitution instance of the proper axiom, **L1**, of S4.04:

L1.2 $(Lp \supset Lq) \supset (LML(Lp \supset Lq) \supset L(Lp \supset Lq))$.

In view of the deductive relations existing between this system and the remaining systems between S4.4 and S4, I came to designate it 'S4.03'. About half a year after the submission of my paper to this Journal, I learned to my surprise, upon studying the July 1977 issue, that the very same name had meanwhile been used by G. N. Georgacarakos in [1] to designate the (equally new) system which results from appending to S4 the following weakening of the proper axiom, **F1**, of S4.3.2:

I1 $L(Lp \supset q) \vee (LMLq \supset p)$.

The aim of this note is to show that Georgacarakos' S4.03 and my S4.03 are one and the same system.

The following straightforward derivation shows that, in the field of S4, **L1.2** inferentially entails **I1**:

(1)	$LMLq \supset LMLLq$	S4°
(2)	$LMLLq \supset LML(Lp \supset Lq)$	S2°
(3)	$\neg p \supset (Lp \supset Lq)$	S1
(4)	$\neg(LMLq \supset p) \supset (LML(Lp \supset Lq) \wedge (Lp \supset Lq))$	(1)-(3), **PC**
(5)	$(Lp \supset Lq) \supset (LML(Lp \supset Lq) \supset L(Lp \supset Lq))$	**L1.2**
(6)	$L(Lp \supset Lq) \supset L(Lp \supset q)$	S1
I1	$(LMLq \supset p) \vee L(Lp \supset q)$	(4)-(6), **PC**

Hence Georgacarakos' S4.03 is included in my S4.03.

For the converse, note that Georgacarakos has shown in [2] that his S4.03 contains exactly those theorems which are validated by all S4-models satisfying the additional requirement that the accessability relation R be "disjunctively symmetrical" which means: "[…] for every $w_i \epsilon W$ there exists a w_j such that $w_i R w_j$ and for any w_k, $w_l \epsilon W$ if $w_i R w_k$ and $w_j R w_l$, then either $w_k R w_i$ or $w_l R w_k$." ([2], p. 504). It can be routinely checked that **L1.2** is validated by these "disjunctively symmetrical" S4-models and is hence a theorem of Georgacarakos' S4.03. That the latter system contains my S4.03 may, however, be even more simply proven by means of the following deduction:

(7)	$Lp \supset ((Lp \supset Lq) \supset Lq)$	**PC**
(8)	$Lq \supset L(Lp \supset Lq)$	S3

(9) $(Lp \supset Lq) \supset (Lp \supset L(Lp \supset Lq))$ (7), (8), **PC**

(10) $L(LLp \supset (Lp \supset Lq)) \vee (LML(Lp \supset Lq) \supset Lp))$ **I1**: p/Lp, $q/Lp \supset Lq$

(11) $L(LLp \supset (Lp \supset Lq)) \supset L(Lp \supset Lq)$ S4°

(12) $LML(Lp \supset Lq) \wedge \neg Lp \supset L(Lp \supset Lq)$ (10), (11), **PC**

L1.2 $(Lp \supset Lq) \supset (LML(Lp \supset Lq) \supset L(Lp \supset Lq))$ (9), (12), **PC**

Hence, in the field of S4, **I1**, and **L1.2** are inferentially equivalent; accordingly, for further investigations, only one S4.03 has to be taken into account.

References

[1] Georgacarakos, G. N., "Additional extensions of S4", *Notre Dame Journal of Formal Logic*, vol. XVIII (1977), pp. 477-488.

[2] Georgacarakos, G. N., "Semantics for S4.03", *Notre Dame Journal of Formal Logic*, vol. XVIII (1977), pp. 504-506.

[3] Lenzen, W., "On some substitution instances of **R1** and **L1**," *Notre Dame Journal of Formal Logic*, vol. XIX (1978), pp. 159-164.