Wolfgang Lenzen

Leibniz on Alethic and Deontic Modal Logic

Summary

This paper is divided in five parts. In section (1) I want to give an overview of the structure of the system of Leibniz's logic. In section (2) I will present the fundamentals of Leibniz's algebra of concepts, *L1*. In section (3) I will show how by means of a simple, ingenious device Leibniz transformed the algebra of concepts, *L1*, into an algebra of propositions, *PL1* (which turns out to represent a system of *strict* implication). In section (4) I will describe how Leibniz developed the basic idea of possible-worlds-semantics for the interpretation of the alethic modal operators 'necessary' 'contingent', 'possible' and 'impossible'. Finally, in section (5) I will argue that Leibniz not only discovered the strict analogy between the logical laws for deontic operators 'forbidden', 'obligatory', and 'allowed' on the one hand and the alethic operators on the other hand; but that he even anticipated A. R. ANDERSON's [1958] idea of „defining" the former in terms of the latter.

1 The structure of the system of Leibniz's logic

Leibniz's main concern in logic was to generalize the traditional theory of the syllogism to a much more general „calculus universalis" which basically consisted of three calculi which I refer to as *L1*, *PL1*, and *L2*. The relation between theses calculi (and some further relations to certain subsystems L0.4 and L0.8) can be displayed in the following diagram:

The System of Leibniz's Logic

L0.4 *Calculus de Continentibus ...*

↓

L0.8 *Plus-Minus-Calculus*

↓

L1 *Algebra of Concepts*

↓ ↘

L2 **PL1** *Calculus of Strict Implication*

This diagram shows five interconnected calculi. Four of them form a chain of increasingly stronger logics *L0.4*, *L0.8*, *L1*, and *L2*, where the decimals are meant to indicate the respective logical strength of the calculus. They are all *concept logics* or term-logics, to use a denotation familiar from the historiography of logic. The 5th calculus, *PL1*, however, is a system of *propositional* logic which can be derived from *L1* by mapping the concepts and conceptual operators into the set of propositions and propositional operators.

The most important calculus, no doubt, is *L1*, the full algebra of concepts that Leibniz developed mainly in the *General Inquiries* of 1686. As has been shown in Lenzen [1984a], this logical system is deductively equivalent or isomorphic to the ordinary algebra of sets. Moreover, Leibniz happened to provide a complete set of axioms for *L1*. Thus, in a way, he discovered the Boolean algebra 160 years before Boole.

Also of interest is the subsystem *L0.8*. Instead of the conceptual operator of negation, it contains the operator of *subtraction* (and some auxiliary operators). Since, furthermore, the conjunction of concepts is symbolised by the sign of addition, this system is often referred to as *Plus-Minus-Calculus*. Leibniz developed it mainly in the famous paper „Non inelegans specimen demonstrandi in abstractis” dating from around 1687. The Plus-Minus-Calculus is inferior to the full algebra in two respects: First, it is conceptually weaker than the latter, i.e. not all operators of *L1* are either present or definable in *L0.8*. Second, unlike in the case of *L1*, the axioms and theorems of the Plus-Minus-Calculus as stated by Leibniz fail to give a complete axiomatization of this logic. By the way, the decimal in the name *L0.8* can be understood to express the degree of conceptual incompleteness – just 80 percent of the operators of *L1* can be handled in the Plus-Minus-Calculus.

In the same sense, the weakest calculus *L0.4* contains only 40 percent of the operators of *L1*. Both the operator of conceptual negation and its substitute, conceptual subtraction, and some other operators depending on these are lacking there. Because of the presence of the main operators of containment and converse containment, i.e. being contained, Leibniz sometimes referred to it as the *Calculus de Continentibus et Contentis*. He began to develop it as early as in 1676; and a final, complete version is already contained in the famous fragment „Specimen Calculi Universalis" together with „Ad Specimen Calculi Universalis Addenda" dating from around 1680. Leibniz re-formulated this calculus some years later in the so-called „Study in the Calculus of Real Addition", i.e. fragment # XX of Vol. 7 of the Gerhardt-edition (**GP**). In view of the fact that the Plus-Calculus *L0.4* is only a weak subsystem of the Plus-Minus-Calculus, *L0.8*, is must appear somewhat surprising that many Leibniz-scholars came to regard the former as superior to the latter.

Now, one characteristic feature of Leibniz's algebra of concepts is that it is in the first instance *based upon* the propositional calculus, but that if afterwards serves as a *basis for* propositional logic. When Leibniz states and proves the laws of concept logic, he takes the requisite rules and laws of propositional logic for granted. Once the former have been established, however, the latter can be obtained from the former ones by observing that there exists a strict analogy between *concepts* and *propositions* which allows one to re-interpret the conceptual connectives as propositional connectives. This seemingly circular procedure which leads from the algebra of concepts, *L1*, to an algebra of propositions, *PL1*, will be explained in some detail in section 3 below. At the moment suffice it to say that in the 19th century George Boole in roughly the same way first presupposed propositional logic to develop his algebra of sets, and only afterwards derived the propositional calculus out of the set-theoretical calculus. Now, while Boole thus arrived at the classical, two-valued propositional calculus, the Leibnizian procedure instead yields a *modal* logic of strict implication. As has been shown in LENZEN [1987], calculus *PL1* is deductively equivalent to the so-called Lewis-modal system S3°.

The final extension of the Leibnizian system is achieved by the theory of „indefinite concepts" which constitutes an ingenious anticipation of the modern theory of 1st and 2nd order quantification. To be sure, Leibniz's theory is somewhat defective and certainly it is far from complete. But his ideas concerning quantifying about concepts and quantifying about individuals (or individual-concepts) were clear and detailed enough to admit of an unambiguous reconstruction. The resulting system *L2* differs from an orthodox (say, Fregean) 2nd order logic in the following respect. While normally one begins by quantifying over individuals on the 1st level and introduces quantification over predicates only on the 2nd level, in the Leibnizian system quantification over *concepts* comes first, and quantifying over individuals is introduced by definition only afterwards. For reasons and space I cannot deal with this interesting system here.[1]

2 Leibniz's Algebra of Concepts (L1) and its Extensional Interpretation

The starting point for Leibniz' universal calculus is the traditional „Aristotelian" theory of the syllogism with its categorical forms of universal or particular, affirmative or negative propositions which express the following relations between two concepts A and B:

U.A.	Every A is B	U.N.	No A is B
P.A.	Some A is B	P.N.	Some A is not B

Within the framework of so-called „Scholastic" syllogistics negative concepts Not-A are also taken into account, which shall here be symbolized as $\overline{A}$. According to the principle of so-called obversion, the U.N. 'No A is B' is equivalent to a corresponding U.A. with the negative predicate: Every A is Not-B. Thus in view of the well-known laws of opposition – according

1 Cf., however, LENZEN [1984a] or chapter 3 of LENZEN [1990].

to which P.N. is the (propositional) negation of U.A. and P.A. is the negation of U.N. – the categorical forms can be represented uniformly as follows:

U.A.	Every A is B	U.N.	Every A is $\overline{\text{B}}$
P.A.	¬(Every A is $\overline{\text{B}}$)	P.N.	¬(Every A is B).

The algebra of concepts as developed by Leibniz in some early fragments of around 1679 and above all in the **GI** of 1686 grows out of this syllogistic framework by three achievements. First, Leibniz drops the expression ‘every’ [‘omne’] and formulates the U.A. simply as ‘A is B’ [‘A est B’] or also as ‘A contains B’ [‘A continet B’]. This fundamental proposition shall here be symbolized as ‚A∈B’, and the negation ¬(A∈B) will be abbreviated as ‚A∉B’. Second, Leibniz introduces the new operator of conceptual *conjunction* which combines two concepts A and B by juxtaposition to AB. Third, Leibniz disregards all traditional restrictions concerning the number of premises and concerning the number of concepts in the premises of a syllogism. Thus arbitrary inferences between sentences of the form A∈B will be taken into account, where the concepts A and B may be arbitrarily complex, i.e. they may contain negations and conjunctions of other concepts. Let the resulting language be referred to as *L1*.

One possible axiomatization of *L1* would take (besides the tacitly presupposed propositional functions ¬, ∧, ∨, →, and ↔) only negation, conjunction and the ∈-relation as primitive conceptual operators. As regards the relation of conceptual containment, A∈B, it is important to observe that Leibniz’s formulation ‘A contains B’ pertains to the so-called *intensional* interpretation of concepts as *ideas*, while we here want to develop an *extensional* interpretation in terms of *sets of individuals*, *viz*. the sets of all individuals that fall under the concepts A and B, respectively. Leibniz explained the mutual relationship between the „intensional” and the extensional point of view in the following passage of the *New Essays on Human understanding*:

> *„The common manner of statement concerns individuals, whereas Aristotle’s refers rather to ideas or universals. For when I say Every man is an animal I mean that all the men are included amongst all*

the animals; but at the same time I mean that the idea of animal is included in the idea of man. ‘Animal’ comprises more individuals than ‘man’ does, but ‘man’ comprises more ideas or more attributes: one has more instances, the other more degrees of reality; one has the greater extension, the other the greater intension” (cf. **GP** 5: 469; my translation).

If ‚Int(A)’ and ‚Ext(A)’ abbreviate the „intension“ and the extension of a concept A, respectively, then the so-called *law of reciprocity* can be formalized as follows:

(RECI 1) $Int(A) \subseteq Int(B) \leftrightarrow Ext(A) \supseteq Ext(B)$.

This principle immediately entails that two concepts have the same ”intension” if and only if they also have the same extension:

(RECI 2) $Int(A) = Int(B) \leftrightarrow Ext(A) = Ext(B)$.

But the latter „law” appears to be patently false! On the basis of our *modern* understanding of intension and extension, there exist many concepts or predicates A, B which have the same extension but which nevertheless differ in intension. Consider, e.g., the famous example in QUINE [1953: 21], A = ’creature with a heart’, B = ’creature with a kidney’, or the more recent observation in SWOYER [1995: 103] (inspired by Quine and directed against RECI 1):

„For example, it might just happen that all cyclists are mathematicians, so that the extension of the concept being a cyclist *is a subset of the extension of the concept* being a mathematician*. But few philosophers would conclude that the concept* being a mathematician *is in any sense included in the concept* being a cyclist*”.*

However, these examples cannot really refute the law of reciprocity *as understood by Leibniz*. For Leibniz, the *extension* of a predicate A is not just the set of all *existing* individuals that (happen to) fall under concept A, but rather the set of all *possible* individuals that have that property. Thus Leibniz would certainly admit that the intension or „idea” of a mathematician is not included in the idea of a cyclist. But he would point out that even if in the *real world* the set of all mathematicians should by chance coincide with the set of all cyclists, there clearly are other possible individuals *in other possible worlds* which are mathematicians and not bicyclists (or bicyclists but not mathematicians). In general, whenever two concepts A and

B differ in intension, then it is possible that there exists an individual which has the one property but not the other. Therefore, given Leibniz's understanding of what constitutes the extension of a concept it follows that A and B differ also in extension.[2]

In LENZEN [1983] precise definitions of the „intension" and the extension of concepts have been developed which satisfy the above law of reciprocity, RECI 1. Leibniz's „intensional" point of view thus becomes provably equivalent, i.e. translatable or transformable into the more common set-theoretical point of view, provided that the extensions of concepts are taken from a universe of discourse, U, to be thought of as a set of *possible individuals*. In particular, the „intensional" proposition $A \in B$, according to which concept A *contains* concept B, has to be interpreted extensionally as saying that the set of all A's *is included* in the set of all B's. The first condition for the definition of an extensional interpretation of the algebra of concepts thus runs as follows:

(DEF 1) Let U be a non-empty set (of possible individuals), and let ϕ be a function such that $\phi(A) \subseteq U$ for each concept-letter A. Then ϕ is an extensional interpretation of Leibniz's concept logic *L1* if

(1) $\phi(A \in B) = \text{true iff } \phi(A) \subseteq \phi(B)$.

Next consider the identity or *coincidence* of two concepts which Leibniz usually symbolizes by the modern sign '=' or by the symbol '∞', but which he sometimes also refers to only informally by speaking of two concepts being the same [idem, eadem]. As stated, e.g., in § 30 **GI**, identity or coincidence can be defined as mutual containment: „That A is B et B is A is the same as that A and B coincide", i.e.:

(DEF 2) $A=B \leftrightarrow_{df} A \in B \wedge B \in A$.

This definition immediately yields the following condition for an extensional interpretation ϕ:

(2) $\phi(A=B) = \text{true iff } \phi(A) = \phi(B)$.

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[2] As regards the ontological scruples against the assumption of merely possible individuals, cf. the famous paper „On What There Is" in QUINE [1953: 1-19] and the critical discussion in LENZEN [1980: 285 sq.].

In most drafts of the „universal calculus”, Leibniz symbolizes the operator of conceptual *conjunction* by mere juxtaposition in the form AB. Only in the context of the Plus-Minus-Calculus he favored the mathematical ‘+’-sign (sometimes also ‘⊕‘) to express the conjunction of A and B. The intended interpretation is straightforward. The extension of AB is the set of all (possible) individuals that fall under both concepts, i. e. which belong to the *intersection* of the extensions of A and of B:

(3) $\phi(AB) = \phi(A)\cap\phi(B)$.

Let it be noted in passing that the crucial condition (1) which reflects the reciprocity of extension and „intension“ would be derivable from conditions (2) and (3) if the relation $\in$ were defined according to § 83 **GI** in terms of conjunction and identity: „Generally, ‘A is B’ is the same as ‘A=AB’” (**P**, 67), i.e. formally:

(DEF 3) $A\in B \leftrightarrow_{df} A=AB$.

For, clearly, a set $\phi(A)$ coincides with the intersection $\phi(A)\cap\phi(B)$ if and only if $\phi(A)$ is a subset of $\phi(B)$! Furthermore, the relation „A is in B” [A inest ipsi B] may simply be defined as the converse of $A\in B$ according to Leibniz’s remark in § 16 **GI**: „[…] ‘A contains B’ or, as Aristotle says, ‘B is in A’”

(DEF 4) $A\iota B \leftrightarrow_{df} B\in A$.

In view of the law of reciprocity, one thus obtains the following condition:

(4) $\phi(A\iota B)$ = true iff $\phi(A) \supseteq \phi(B)$.

The next element of the algebra of concepts – and, by the way, one with which Leibniz had notorious difficulties – is *negation*. Leibniz usually expressed the negation of a concept by means of the same word he also used to express propositional negation, viz. ‘not’ [non]. Especially throughout the **GI**, the statement that one concept, A, contains the negation of another concept, B, is expressed as ‘A is not-B’ [A est non B], while the related phrase ‘A isn’t B’ [A non est B] has to be understood as the mere negation of ‘A contains B’. As was shown in LENZEN [1986], during the whole period of the development of the „universal calculus" Leibniz had to struggle hard to grasp the important difference between 'A is not-B' and 'A isn't B'. Again and again he mistakenly identified both statements, although he had noted their non-equivalence repeatedly in other places. Here the negation of concept A will be expressed as '$\overline{A}$', while propositional negation is symbolized by means of the usual sign '¬'. Thus 'A is not-B' must be formulated as '$A \in \overline{B}$', while 'A isn't B' has to be rendered as '$\neg A \in B$' or '$A \notin B$'. The intended extensional interpretation of $\overline{A}$ is just the set-theoretical complement of the extension of A, because each individual which fails to fall under concept A eo ipso falls under the negative concept $\overline{A}$:

(5) $\phi(\overline{A}) = \overline{\phi(A)}$.

Closely related with the operator of negation is that of *possibility* or self-consistency of concepts. Leibniz expresses it in various ways. He often says 'A is possible' [A est possibile] or 'A is [a] being' [A est Ens] or also 'A is a thing' [A est Res]. Sometimes the self-consistency of A is also expressed elliptically by 'A est', i.e. 'A is'. Here the capital letter '**P**' will be used to abbreviate the possibility of a concept A, while the impossibility or inconsistency of A shall be symbolized by '**I**(A)'. According to **GI**, lines 330-331, the operator **P** can be defined as follows: „*A* not-*A* is a contradiction. *Possible* is what does not contain a contradiction or *A* not-*A*":

(DEF 5) $\mathbf{P}(B) \leftrightarrow_{df} B \notin A\overline{A}$.[3]

It then follows from our earlier conditions (1), (3), and (4) that **P**(A) is true (under the extensional interpretation ϕ) if and only if $\phi(A)$ is not empty:

(6) $\phi(\mathbf{P}(A))$ = true iff $\phi(A) \neq \emptyset$.

At first sight, this condition might appear inadequate, since there are certain concepts – such as that of a unicorn – which happen to be empty but which may nevertheless be regarded as

[3] This definition might be simplified as follows: $\mathbf{P}(B) \leftrightarrow_{df} B \notin \overline{B}$.

possible, i.e. not involving a contradiction. Remember, however, that the universe of discourse underlying the extensional interpretation of *L1* does not consist of *actually existing* objects only, but instead comprises all *possible* individuals. Therefore the non-emptiness of the extension of A is both *necessary and sufficient* for guaranteeing the self-consistency of A. Clearly, if A is possible then there must exist at least one *possible* individual that falls under concept A.

The main elements of Leibniz's algebra of concepts may thus be summarized in the following diagram.

Element of L1	*Symbolization*	*Leibniz's Notation*	*Set-theoretical Interpretation*
Identity	A=B	A∞B; A=B; coincidunt A et B; ...	$\phi(A) = \phi(B)$
Containment	A∈B	A est B; A continet B	$\phi(A) \subseteq \phi(B)$
Converse Containment	AιB	A inest ipsi B	$\phi(A) \supseteq \phi(B)$
Conjunction	AB	AB; A+B	$\phi(A) \cap \phi(B)$
Negation	$\overline{A}$	Non-A	$\overline{\phi(A)}$
Possibility	**P**(A)	A est Ens; A est res; A est possibile	$\phi(A) \neq \emptyset$

Let's now have a brief look at some *axioms* and *theorems* of *L1*! The subsequent selection of principles, all of which (with the possible exception of the last one) have been stated by Leibniz himself, is more than sufficient to derive the laws of the Boolean algebra of sets:

Theorems of L1	*Formal version*	*Leibniz's version*
CONT 1	A∈A	„B is B" (**GI**, § 37)
CONT 2	A∈B ∧ B∈C → A∈C	„[...] if A is B and B is C, A will be C" (**GI**, § 19)
CONT 3	A∈B ↔ A=AB	„Generally '*A* is *B*' is the same as '*A* = *AB*'" (**GI**, § 83)

CONJ 1	$A \in BC \leftrightarrow A \in B \wedge A \in C$	„That A contains B and A contains C is the same as that A contains BC" (**GI**, § 35; cf. **P** 58, note 4)
CONJ 2	$AB \in A$	„AB is A" (**C,** 263)
CONJ 3	$AB \in B$	„AB is B" (**GI**, § 38)
CONJ 4	$AA = A$	„AA = A" (**GI**, § 171, Third)
CONJ 5	$AB = BA$	„AB ∞ BA" (**C**. 235, # (7))
NEG 1	$\overline{\overline{A}} = A$	„Not-not-A = A" (**GI**, § 96)
NEG 2	$A \neq \overline{A}$	„A proposition false in itself is 'A coincides with not-A'" (**GI**, § 11)
NEG 3	$A \in B \leftrightarrow \overline{B} \in \overline{A}$	„In general, 'A is B' is the same as 'Not-B is not-A'" (**GI**, § 77)
NEG 4	$\overline{A} \in \overline{AB}$	„Not-A is not-AB" (**GI**, § 76a)
NEG 5	$[\mathbf{P}(A) \wedge]\ A \in B \rightarrow A \notin \overline{B}$	„If A is B, therefore A is not not-B" (**GI**, § 91)
POSS 1	$\mathbf{I}(A\overline{B}) \leftrightarrow A \in B$	„if I say 'A not-B is not', this is the same as if I were to say [...]'A contains B'" (**GI**, § 200).[4]
POSS 2	$A \in B \wedge \mathbf{P}(A) \rightarrow \mathbf{P}(B)$	„If A contains B and A is true, B is also true" (**GI**, § 55)[5]
POSS 3	$\mathbf{I}(A\overline{A})$	„A not-A is not a thing" (**GI**, § 171, Eighth)
POSS 4	$A\overline{A} \in B$	„[...] the round square is a quadrangle with null-angles. For this proposition is true in virtue of an impossible hypothesis" (**GP** 7, 224/5)[6]

CONT 1 and CONT 2 show that the relation of *containment* is reflexive and transitive: Every concept contains itself; and if A contains B which in turn contains C, then A also contains C. CONT 3 shows that the fundamental relation $A \in B$ might be defined in terms of conceptual conjunction (plus identity).

CONJ 1 is the decisive characteristic axiom for *conjunction*, and it establishes a connection between *conceptual* conjunction on the one hand and *propositional* conjunction on

4 Parkinson translates Leibniz's „Si dicam AB non est ..." somewhat infelicitous as „If I say ‚AB does not exist'..." thus blurring the distinction between (actual) existence and mere possibility. For an alternative formulation of Poss 1 cf. **C**., 407/8: „[…] si A est B vera propositio est, A non-B implicare contradictionem", i.e. 'A is B' is a true proposition if A non-B includes a contradiction.

5 At first sight this quotation might seem to express some law of propositional logic such as *modus ponens*: If A→B and A, then B. However, as Leibniz goes on to explain, when applied to concepts, a „true" term is to be understood as one that is self-consistent: „[...] By ‚a false letter' I understand either a false term (i.e. one which is impossible, or, is a non-entity) or a false proposition. In the same way ‚true' can be understood as either a possible term or a true proposition" (ibid.). As to the contraposited form of POSS 2, $A \in B \wedge \mathbf{I}(B) \rightarrow \mathbf{I}(A)$, cf. also the special case in **C**., 310: „Et sanè si DB est non Ens [...] etiam CDB erit non ens".

6 As the text-critical apparatus in **A** VI, 4, 293 reveals, Leibniz had originally added: „Nimirum de impossibile concluditur impossibile". So in a certain way he was aware of the principle „ex contradictorio quodlibet" according to which not only a contradictory *proposition* logically entails any arbitrary proposition, but also a contradictory or „impossible" *concept* contains any other concept.

the other: Concept A contains 'B and C' iff A contains B and A also contains C. The remaining theorems CONJ 2 - CONJ 5 may be derived from CONJ 1 with the help of corresponding truth-functional tautologies.

Negation is axiomatized by means of three principles, the law of double negation NEG 1, the law of consistency NEG 2, which says that every concepts differs from its own negation, and the well known principle of contraposition, NEG 3, according to which concept A contains concept B iff $\overline{B}$ contains $\overline{A}$. The further theorem NEG 4 may be obtained from NEG 3 in virtue of CONJ 2.

The important principle POSS 1 says that concept A contains concept B iff the conjunctive concept A Not-B is impossible. This principle also characterizes negation, though only indirectly, since according to DEF 4 the operator of *self-consistency* of concepts is definable in terms of negation and conjunction. POSS 2 says that a term B which is contained in a self-consistent term A will itself be self-consistent. POSS 3 easily follows from POSS 1 in virtue of CONT 1. POSS 4 is the counterpart of what one calls „ex contradictorio quodlibet" in propositional logic: An inconsistent concept contains every other concept! This law was not explicitly stated by Leibniz but it may be regarded as a genuinely Leibnizian theorem because it follows from POSS 1 and POSS 3 in conjunction with the observation that, since $A\overline{A}$ is inconsistent, so is, according to POSS 2, also $A\overline{A}\ \overline{B}$.

As was shown in LENZEN [1984b: 200], the set of principles {CONT 1, CONT 2, CONJ 1, NEG 1, POSS 1, POSS 2} already provides a complete axiomatization of the algebra of concepts which is isomorphic to the Boolean algebra of sets.

3 Leibniz's Calculus of Strict Implication

Although Leibniz never spent much time for the investigation of the proper laws of *propositional* logic, he must yet be credited with the following discovery of utmost importance. He devised a simple, but ingenious method to transform the algebra of concepts into an algebra of propositions. Already in the fragment *Notationes Generales*, probably written between 1683 and 1685[7], Leibniz pointed out to the parallel between the containment relation among concepts and the implication relation among propositions. Just as the simple proposition 'A is B' (where A is the „subject", B the „predicate") is true, „when the predicate is contained in the subject", so a conditional proposition 'If A is B, then C is D' (where 'A is B' is designated as 'antecedent', 'C is D' as 'consequent') is true, „when the consequent is contained in the antecedent" (cf. **A** VI, 4, 551). In later works Leibniz compressed this idea into formulations such as „a proposition is true whose predicate is contained in the subject *or more generally* whose consequent is contained in the antecedent".[8] The most detailed explanation of the basic idea of deriving the laws of the algebra of *propositions* from the laws of the algebra of *concepts* was sketched in §§ 75, 137 and 189 **GI** as follows:

> *„If, as I hope, I can conceive all propositions as terms, and hypotheticals as categoricals [...] this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance" […]*
>
> *„We have, then, discovered many secrets of great importance for the analysis of all our thoughts and for the discovery and proof of truths. We have discovered [...] how absolute and hypothetical truths have one and the same laws and are contained in the same general theorems" […]*
>
> *„Our principles, therefore, will be these [...] Sixth, whatever is said of a term which contains a term can also be said of a proposition from which another proposition follows" (**P** 66, 78, 85).*

To conceive all propositions in analogy to concepts („instar terminorum") means in particular that the hypothetical proposition 'If α then β' will be logically treated exactly like the fundamental relation of containment between concepts, 'A contains B'. Furthermore, as Leibniz explained elsewhere, negations (and conjunctions) of propositions are to be conceived just as negations (and conjunctions) of concepts:

7 Cf. **A** VI, 4, # 131.

8 Cf. **C**. 401: „vera autem propositio est cujus praedicatum continetur in subjecto, *vel generalius* cujus consequens continetur in antecedente" (my emphasis); cf. also **C**. 518: „Semper igitur praedicatum seu consequens inest subjecto seu antecedenti".

„If A is a proposition or statement, by non-A I understand the proposition A to be false. And if I say ‘A is B’, and A and B are propositions, then I take this to mean that B follows from A […] This will also be useful for the abbreviation of proofs; thus if for ‘L is A’ we would say ‘C’ and for ‘L is B’ we say ‘D’, then for this [hypothetical] ‘If L is B, it follows that L is B’ one could substitute ‘C is D’.”[9]

One thus obtains the following „mapping” of the primitive formulas of the algebra of concepts into primitive formulae of an algebra of propositions:

$A \in B$	$\alpha \rightarrow \beta$
$\overline{A}$	$\neg\alpha$
AB	$\alpha\wedge\beta$

As Leibniz himself mentioned, the fundamental law POSS 1 does not only hold for the containment-relation between concepts but equally for the entailment relation between propositions:

„A contains B is a true proposition if A non-B entails a contradiction. This applies both to categorical and to hypothetical propositions, e.g., ‘If A contains B, C contains D’ can be formulated as follows: ‘That A contains B contains that C contains D’; therefore ‘A containing B and at the same time C not containing D’ entails a contradiction.”[10]

Hence $A \in B \leftrightarrow \mathbf{I}(A\overline{B})$ may be „translated” into $(\alpha\rightarrow\beta) \leftrightarrow \neg\Diamond(\alpha\wedge\neg\beta)$. This formula shows that Leibniz’s implication is not a material but rather a *strict* implication. As was already noted by RESCHER [1954: 10], Leibniz’s account provides a definition of „entailment in terms of negation, conjunction, and the notion of possibility”, for α implies β iff it is *impossible* that

9 Cf. **C**., 260, # 16: „Si A sit propositio vel enuntiatio, per non-A intelligo propositionem A esse falsam. Et cum dico A est B, et A et B sunt propositiones, intelligo ex A sequi B. […] Utile etiam hoc ad compendiose demonstrandum, ut si pro L est A dixissemus C et pro L est B dixissemus D pro ista si L est A sequitur quod L est B, substitui potuisset C est D.”

10 Cf. **C**., 407: „*Vera propositio* est A continet B, si A non-B infert contradictionem. *Comprehenduntur et categoricae et hypotheticae propositiones*, v.g. si A continet B, C continet D, potest sic formari: A continere B continet C continere D; itaque A continere B, et simul C non continere D infert contradictionem” (second emphasis is mine).

α is true while β is false. This definition of strict implication which was to be „re-invented", e. g., by C. I. Lewis[11] had been formulated also in the „Analysis Particularum":

> *„Thus if I say 'If L is true it follows that M is true', this means that one cannot suppose at the same time that L is true and that M is false".*[12]

As regards the other non-primitive elements of *L1*, the relation 'A is in B' represents, according to DEF 4, the converse of $A \in B$. Hence its propositional counterpart is the „inverse implication", $\alpha \leftarrow \beta$. According to DEF 2, the coincidence relation A=B is tantamount to mutual containment, $A \in B \wedge B \in A$, which will thus be translated into a mutual implication between propositions, $(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$, i.e. into the strict equivalence, $\alpha \leftrightarrow \beta$. Next, according to DEF 5, the possibility or self-consistency of a concept B amounts to the conditions $B \notin A\overline{A}$. In the field of propositions one hence obtains that α is possible, $\Diamond\alpha$, if and only if α does not entail a contradiction: $\neg(\alpha \rightarrow (\beta \wedge \neg\beta))$.

AιB	$(\alpha \leftarrow \beta)$	$[\leftrightarrow_{df} (\beta \rightarrow \alpha)]$
A=B	$\alpha \leftrightarrow \beta$	$[\leftrightarrow_{df} (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)]$
P(A)	$\Diamond\alpha$	$[\leftrightarrow_{df} \neg(\alpha \rightarrow (\beta \wedge \neg\beta))]$

Given this „translation", the basic axioms and theorems of the algebra of concepts listed in section 2 may be transformed into the following set of laws of an algebra of propositions:

	Basic Principles of PL1
IMPL 1	$(\alpha \rightarrow \alpha)$
IMPL 2	$((\alpha \rightarrow \beta) \wedge (\beta \rightarrow \gamma)) \rightarrow (\alpha \rightarrow \gamma)$
IMPL 3	$(\alpha \rightarrow \beta) \leftrightarrow (\alpha \leftrightarrow \alpha \wedge \beta)$

11 Cf. e.g., LEWIS & LANGFORD [1932: 124]: „The relation of strict implication can be defined in terms of negation, possibility, and product [...] Thus „p implies q" [...] is to mean „It is false that it is possible that p should be true and q false".

12 Cf. **A** VI, 4, 656: „Itaque si dico *Si L est vera sequitur quod M est vera*, sensus est, non simul supponi potest quod *L* est vera, et quod *M* est falsa'".

CONJ 1	$(\alpha \to \beta\wedge\gamma) \leftrightarrow ((\alpha\to\beta) \wedge (\alpha\to\gamma))$
CONJ 2	$\alpha\wedge\beta \to \alpha$
CONJ 3	$\alpha\wedge\beta \to \beta$
CONJ 4	$\alpha\wedge\alpha \leftrightarrow \alpha$
CONJ 5	$\alpha\wedge\beta \leftrightarrow \beta\wedge\alpha$
NEG 1	$(\neg\neg\alpha \leftrightarrow \alpha)$
NEG 2	$\neg(\alpha \leftrightarrow \neg\alpha)$
NEG 3	$(\alpha \to \beta) \leftrightarrow (\neg\beta\to \neg\alpha)$
NEG 4	$\neg\alpha \to \neg(\alpha\wedge\beta)$
NEG 5	$[\Diamond\alpha \wedge] (\alpha \to \beta) \to \neg (\alpha \to \neg\beta)$
POSS 1	$(\alpha \to \beta) \leftrightarrow \neg\Diamond(\alpha \wedge \neg\beta)$
POSS 2	$(\alpha \to \beta) \wedge \Diamond\alpha \to \Diamond\beta$
POSS 3	$\neg\Diamond(\alpha \wedge \neg\alpha)$
POSS 4	$(\alpha\wedge\neg\alpha) \to \beta$

Although Leibniz didn't care very much about propositional logic, he happened to put forward at least some of these laws in scattered fragments. For instance, in the first juridical disputation *De Conditionibus* the transitivity of the inference relation, IMPL 2, was characterized as follows: „The Co[ndition] of the co[ndition] is the co[ndition] of the co[nditioned]. If by positing A B will be posited and by positing B C will be posited, then also by positing A C will be posited".[13] As regards IMPL 1 and CONJ 2, 3, Leibniz mentions in the fragment „De Calculo Analytico Generale" the „Primary Consequences: A is B, therefore A is B […] A is B and C is D, therefore A is B, or as well [therefore] C is D", and the

[13] Cf. **A** VI, 1, 110: „C[onditi]o C[onditio]nis est C[onditi]o C[onditiona]ti. Si posito A positur B, et posito B positur C; etiam posito A positur C." For a discussion of Leibniz's early work on juridic (or deontic) logic cf. SCHEPERS [1975].

corresponding „Axioms […] 3) If A is B, also A is B. If A is B and B is C, also A is B". Furthermore the definition of strict implication in terms of strict equivalence plus conjunction, IMPL 3, was exemplified in another fragment as follows:

> *„A true hypothetical proposition of first degree is 'If A is B, and from this it follows that C is D' […] Let the state of affairs 'A is B' be called L, and the state of affairs 'C is D' be called M. Then one obtains L = LM; in this way the hypothetical [proposition] is reduced to a categorical" (cf.* ***C****. 408).*

Moreover in „De Varietatibus Enuntiationum" Leibniz put forward principle CONJ 1 for the special case A = 'a is b', B = 'e is d' and C = 'l is m' by maintaining that the proposition „If a is b it follows that e is d and l is m" can be resolved into the conjunction of the propositions „If a is b it follows that e is d" and „If a is b it follows that l is m" (cf. **A** VI, 4, 129). Versions of the principle of double negation, NEG 1, may be found in § 4 **GI** or, for the special cases of propositions of the type 'A=B' and 'A∈B', more formally in **C**. 235[14]. Finally the „Analysis particularum" contains besides the above quoted paraphrase of POSS 1 also the law of (propositional) contraposition NEG 3: „If a proposition M […] follows from a proposition L […], then conversely the falsity of the proposition L follows from the falsity of the proposition M".

The above collection of basic principles does not yet, however, constitute a genuine *calculus* of (modal) propositional logic. At least some additional *rules of deduction* are needed which allow one to derive further theorems from these „axioms". As was shown in LENZEN [1987], Leibniz was well aware of the validity of the rule of (strict) *modus ponens*:

(MP) $(\alpha\rightarrow\beta), \alpha \vdash \beta$

and of the rule of conjunction:

(RC) $\alpha, \beta \vdash \alpha \wedge \beta$.

Furthermore it was argued there that the mapping of *L1* into *PL1* yields a calculus of strict implication in the vicinity of Lewis' system S2°. This does not mean, however, that Leibniz

14 „Idem sunt A ∞ B [...] et A non non ∞ B"; cf. also **C**. 262: „A non non est B, idem est quod A est B"

would have favored such a weak system as the proper calculus of (alethic) modal logic. For example, Leibniz would certainly have subscribed to the validity of the truth-axiom $\alpha \rightarrow \alpha$ (or, equivalently, $\alpha \rightarrow \Diamond\alpha$). But, for purely syntactical reasons, these laws can never be obtained from corresponding theorems of *L1*by way of Leibniz's consideration of propositions „instar terminorum".[15] For reasons of space, this issue shall not be discussed here further – the reader is referred to the detailed exposition in LENZEN [1987]. Only a few more theorems for the modal operators und $\Diamond$ shall be considered in the subsequent section where Leibniz's version of a possible worlds semantics is represented.

4 Leibniz's Possible Worlds Semantics

The fundamental logical relations between necessity, , possibility, $\Diamond$, and impossibility can be expressed, e.g., by:

(NEC 1) $(\alpha) \leftrightarrow \neg\Diamond(\neg\alpha)$

(NEC 2) $\neg\Diamond(\alpha) \leftrightarrow (\neg\alpha)$.

Of course, these laws were familar already to logicians long before Leibniz. However, Leibniz not only formulated, e.g., NEC 1 already as a youth, at the age of 25, as follows:

> *„Whenever the question is about necessity, the question is also about possibility, for if something is called necessary, then the possibility of its opposite is negated"*[16]

but he also „proved“ these relations by means of an admirably clear semantic analysis of modal operators in terms of „possible cases", i.e. possible worlds:

„Possible	*is whatever*	*can*	*happen*	*or what is true*	*in some cases*
Impossible	*is whatever*	*cannot*	*happen*	*or what is true*	*in no […] case*
Necessary	*is whatever*	*cannot*	*not happen*	*or what is true*	*in every […] case*
Contingent	*is whatever*	*can*	*not happen*	*or what is [not] true in some case".*[17]	

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[15] E.g., $\alpha \rightarrow \Diamond\alpha$ could only result from mapping the formula $A \in \mathbf{P}(A)$ or $A \rightarrow \mathbf{P}(A)$ into *PL1*; but none of these formulae is syntactically well-formed!

[16] Cf. **A** VI, 1, 460: „Quoties autem de necessitate quaestio est, de possibilitate quaestio est, nam quid necessarium dicitur, possibilitas oppositi negatur".

Hence a proposition α is *possible* iff α is true in at least one case; α is *impossible*, iff α is true in no case; α is *necessary* iff α is true in each case; and, finally, α is *contingent*, i.e. non-necessary, iff α is not true in at least one case.[18] Now this analysis of the truth-conditions for modal propositions not only entails the above mentioned laws NEC 1 and 2, but it also gives rise to the principle that whenever α is necessary, α will be possible as well, and by contraposition: „Because all that is necessary is possible, all that is impossible is contingent”:[19]

(NEC 3) $\alpha \rightarrow \Diamond(\alpha)$,

(NEC 4) $\neg\Diamond(\alpha) \rightarrow \neg\ (\alpha)$.

Leibniz „demonstrates” these laws by reducing them to corresponding laws for (universal and existential) quantifiers such as: „If α is true in each case, then α is true in at least one case”. These quantificational principles were tacitly presupposed by Leibniz who only mentioned them in passing by maintaining (very elliptically), e.g.: „‘All’ is the same as ‘none not’” or „‘All not’ is the same as ‘none’”. Cf. the following „proof” of NEC 2:

> *„[…] ‘necessarily not happen’ and ‘impossible’ coincide. For also ‘none’ and ‘everything not’ coincide. Why so? Because ‘none’ is ‘not something’. ‘Every’ is ‘not something not’. Therefore ‘everything not’ is ‘not something not not’. The two latter ‘not’ destroy each other, thus remains ‘not something’.”*

On the background of certain rules for the negation of the quantifier expressions ‘all’, ‘some’, and ‘none’, which reflect the core ideas of the traditional theory of opposition of categorical forms, Leibniz thus argues that an impossible proposition which is false in every case is the

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17 Cf. **A** VI, 1, 466:

„Possibile	est quicquid	potest	fieri seu quod verum est	quodam	casu
Impossibile	est quicquid	non potest	fieri seu quod verum est	nullo [...]	casu
Necessarium	est quicquid	non potest non	fieri seu quod verum est	omni [...]	casu
Contingens	est quicquid	potest non	fieri Seu quod verum est	quodam non	casu.“

18 As this quotation shows, Leibniz uses the notion of contingency not in the modern sense of ‚neither necessary nor impossible’ but as the simple negation of ‚necessary’.

19 Cf. **A** VI, 4, 2759: „Quia omne necessarium est possibile omne impossibile est contingens seu potest non fieri“.

same as a proposition which is not true in any case. Let it be mentioned in passing that the analogue „proof" of NEC 3 contains a minor mistake which is quite typical of Leibniz[20]:

> *„[…] everything which is necessary is possible. For always, when 'everything is', also 'something is' [the case]. Thus if 'everything is', 'not something is not', or 'something is not not'. Hence 'something is'".*[21]

To be sure, a necessary proposition α which is true in every case, a fortiori has to be true in at least one case, hence α is possible. But this principle - or the corresponding quantificational law $(\forall x\alpha \rightarrow \exists x\alpha)$ – cannot be correctly derived from the presupposed equivalence $(\forall x\alpha \leftrightarrow \neg\exists x\neg\alpha)$ plus the law of double negation, $(\neg\neg\alpha \leftrightarrow \alpha)$ in the way attempted by Leibniz. For 'not something is not', i.e. $\neg\exists x\neg\alpha$, is not the same as 'something is not not', i.e. $\exists x\neg\neg\alpha$!

It cannot be overlooked, however, that the truth conditions quoted from the early *De Conditionibus*, even when combined with Leibniz's later views on possible worlds, fail to come up to the standards of modern possible worlds semantics, since in Leibniz's work nothing corresponds to the accessibility relation among worlds. Therefore it is almost impossible to decide which of the diverse modern systems like T, S4, S5, etc. best conforms with Leibniz's views. According to POSER [1969], Leibniz's modal logic is tantamount to S5. This means in particular that Leibniz accepted the characteristic axiom of S4:

(NEC 5) $\alpha \rightarrow \quad \alpha$.

Poser pointed out to the following passage in „De Affectibus": „For what can impossibly be actually the case, that can impossibly be possible"[22] which rather convincingly shows that, in Leibniz's view, any impossible proposition is impossibly possible:

(NEC 6) $\neg\Diamond\alpha \rightarrow \neg\Diamond\Diamond\alpha$.

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[20] In so far as, again and again, Leibniz had serious problems in distinguishing ‚non est' and ‚est non'; cf. LENZEN [1986].

[21] Cf. **A** VI, 1, 469: „[...] omne necessarium est possibile. Nam semper, si omnis est, etiam quidam est. Si enim Omnis est, non quidam non est seu quidam non non est. Ergo quidam est".

[22] Cf. **Grua**, 534: „Nam quod impossibile est esse actu, id impossibile est esse possibile".

However, Poser failed to give any quotation (or any other compelling reason) to show that Leibniz would also have accepted the stronger S5-principle $\Diamond\alpha \rightarrow \quad \Diamond\alpha$, according to which any possible proposition would be necessarily possible. Moreover, as was argued by ADAMS [1982], the latter principle appears to be incompatible with Leibniz's philosophical view of necessity as expressed, e.g., in the **GI**:

> *„(133) A true necessary proposition can be proved by reduction to identical propositions, or by reduction of its opposite to contradictory propositions; hence its opposite is called ‚impossible'.*
>
> *(134) A true contingent proposition cannot be reduced to identical propositions, but is proved by showing that if the analysis is continued further and further, it constantly approaches identical propositions, but never reaches them." (**P**, 77).*

If a *necessary* proposition α can be reduced in finitely many steps to an „identity", this means that a proposition α is *possible* if and only if it is not refutable in finitely many steps (i.e. its negation cannot be reduced in finitely many steps to an „identity"). But on this understanding of possibility and necessity, the S5 principle $\Diamond\alpha \rightarrow \quad \Diamond\alpha$ appears to be blatantly false.

5 Leibniz's Deontic logic

Leibniz saw very clearly that the logical relations between the „Modalia Iuris" *obligatory*, *permitted* and *forbidden* exactly mirror the corresponding relations between the alethic modal operators *necessary*, *possible* and *impossible* and that therefore all laws and rules of alethic modal logic may be applied to deontic logic as well:

> *„Just like ‘necessary’, ‘contingent’, ‘possible’ and ‘impossible’ are related to each other, so also ‘obligatory’, ‘not obligatory’, ‘permitted’, and ‘forbidden’".*[23]

This structural analogy rests on the important discovery that the deontic notions can be defined by means of the alethic notions plus the additional „logical" constant of a morally perfect man [„vir bonus"]. Such a „virtuous man", *b*, is characterized by the requirements that

[23] Cf. **A** VI, 4, 2762: „Uti se habent inter se necessarium, contingens, possibile, impossibile; ita se habent debitum, indebitum, licitum, illicitum".

(1) *b* strictly obeys all laws, (2) *b* always acts in such a way that he does no harm to anybody, and (3) *b* loves or is benevolent to all other people.[24] Given this understanding of the „vir bonus", *b*, Leibniz explains:

„Obligatory is	*what is*	*necessary*	*for the virtuous man as such*
not obligatory is	*what is*	*contingent*	*for the virtuous man as such*
permitted is	*what is*	*possible*	*for the virtuous man as such*
forbidden is	*what is*	*impossible*	*for the virtuous man as such."*[25]

If we express the restriction of the modal operators and ◊ to the virtuous man by means of a subscript ‚*b*', these definitions can be formalized as follows:

(DEON 1) $\mathbf{O}(\alpha) \leftrightarrow {}_b(\alpha)$

(DEON 2) $\mathbf{E}(\alpha) \leftrightarrow \Diamond_b(\alpha)$[26]

(DEON 3) $\mathbf{F}(\alpha) \leftrightarrow \neg\Diamond_b(\alpha)$

Now, as Leibniz mentioned in passing, all that is unconditionally necessary will also be necessary for the virtuous man as such:[27]

(NEC 7) $(\alpha) \rightarrow {}_b(\alpha)$.

Hence the fundamental laws for the deontic operators can be derived from corresponding laws of the alethic modal operators in much the same way as ANDERSON [1958] reduced deontic logic to alethic modal logic. As Leibniz pointed out, two different classes of theorems may be

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24 Cf. **A** VI, 1, 466: „Vir bonus est quisquis amat omnes"; **A** VI, 4, 2851: „Vir bonus est qui benevolus est erga omnes" and **A** VI, 4, 2856: „Vir bonus censetur, qui hoc agit ut prosit omnibus noceat[que] nulli." It is interesting to note that Leibniz denotes the entire discipline of jurisprudence as the „science of the virtuous man" („scientia viri boni") and justice as the „voluntas viri boni".

25 Cf. **A** VI, 4, 2758:

„Debitum	est, quod viro bono qua tali	necessarium
Indebitum	est, quod viro bono qua tali	contingens
Licitum	est, quod viro bono qua tali	possibile
Illicitum	est, quod viro bono qua tali	impossibile."

In the former edition in **Grua** 605 ‘debitum' was mistakenly associated with ‘contingens'. Cf. also **A** VI, 4, 2863: „quod Viro bono possibile, impossibile, necessarium est, si nomen suum tueri velit, id justum sive licitum, injustum, ac denique debitum esse."

26 We here use the letter ‚E' (reminding of the German ‚erlaubt') instead of ‚P' for ‚permitted' in order to avoid any confusions with the operator for the possibility (or self-consistency) of concepts!

27 Cf. **A** VI, 4, 2759: „Nam omne necessarium est necessarium viro bono".

distinguished. First we have „Theorems in which the juridic modalities are combined by themselves”, i.e. theorems describing the logical relations among the deontic operators, e.g.:

> *„Everything which is obligatory is permitted […] Everything which is forbidden is not obligatory […] Nothing which is obligatory is forbidden […] Nothing which is forbidden is obligatory […] Everything that is forbidden is obligatory to omit. And everything that is obligatory to omit is forbidden. […] Everything that is forbidden to omit is obligatory and everything which is obligatory is forbidden to omit […] Everything which is not obligatory is permitted to omit and everything that is permitted to omit is not obligatory”.*

(DEON 4a) $O(\alpha) \rightarrow E(\alpha)$

(DEON 4b) $\neg E(\alpha) \rightarrow \neg O(\alpha)$

(DEON 5a) $O(\alpha) \rightarrow \neg F(\alpha)$

(DEON 5b) $F(\alpha) \rightarrow \neg O(\alpha)$

(DEON 6) $F(\alpha) \leftrightarrow O(\neg\alpha)$

(DEON 7) $O(\alpha) \leftrightarrow F(\neg\alpha)$

(DEON 8) $\neg O(\alpha) \leftrightarrow E(\neg\alpha)$

As Leibniz „demonstrates” (or, at least, makes plausible), these laws are immediate counterparts of the well-known logical relations between the alethic modalitis. E.g., concerning DEON 6 he remarks:

> *„Everything which is forbidden is obligatory to omit. And everything that is obligatory to omit is forbidden, i.e. ‘forbidden’ and ‘obligatory to omit’ coincide. Because ‘necessarily not happen’ and ‘impossible’ coincide. For also ‘none’ and ‘everything not’ coincide”. (Cf. **A** VI, 1, 469).*

As a second class of theorems one obtains certain „Theorems in which the juridic modalities are combined with the logical modalities”. Thus in the „Elementa Juris Naturalis” Leibniz mentions the following principles concerning the relations between the alethic concepts ‘necessary’, ‘possible’ and ‘impossible’ on the one hand and the deontic notions ‘obligatory, ‘permitted’ and ‘forbidden’ on the other hand: „Everything which is necessary is obligatory”, or, by contraposition: „Everything that is not obligatory is not necessary but contingent”[28]:

(DEON 9a) $(\alpha) \rightarrow O(\alpha)$

(DEON 9b) $\neg O(\alpha) \rightarrow \neg\ (\alpha)$

Furthermore: „Everything that is necessary is permitted”, or, again by contraposition, „Everything that is forbidden is not necessary but contingent” (*ibid*.):

(DEON 10a) $(\alpha) \rightarrow E(\alpha)$

(DEON 10b) $\neg E(\alpha) \rightarrow \neg\ (\alpha)$

Next, „Everything that is permitted is possible”, or „Everything that is impossible is not permitted” (*ibid*.):

(DEON 11a) $E(\alpha) \rightarrow \Diamond(\alpha)$

(DEON 11b) $\neg\Diamond(\alpha) \rightarrow \neg E(\alpha)$

Finally, „Everything which is obligatory is possible”, or „Everything which is impossible is not obligatory, i.e. may be omitted by the virtuous man”[29]:

(DEON 12a) $O(\alpha) \rightarrow \Diamond(\alpha)$

(DEON 12b) $\neg\Diamond(\alpha) \rightarrow \neg O(\alpha)$

To illustrate Leibniz’s way of demonstrating these laws in „Modalia et Elementa Juris Naturalis” let us consider DEON 10a which is formulated there with the word ‚licitum’ instead of ‚justum’ expressing ‘permitted’:

> *„Everything which is necessary is permitted, i.e. necessity has no law.*
>
> *For everything which is necessary is necessary for the virtuous man. If something is necessary for the virtuous man, its opposite is impossible for the virtuous man. What is impossible for the virtuous man is anyway not possible for the virtuous man as such, i.e. it is not permitted. Therefore the opposite of*

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28 Cf. **A** VI, 1, 470: „Omne indebitum nec necessarium est, sed contingens“

29 Cf. **A** VI, 1, 470: „Omne impossibile indebitum seu omissibile est viro bono“.

something necessary is not permitted. However, if the opposite of something is not permitted, then itself is permitted."[30]

By means of the „bridge principle" NEC 7, (α) is first shown to entail $_b(\alpha)$. Next Leibniz makes use of the following law NEC 8 which relativizes the usual equivalence NEC 1 to the „virtuous man":

(NEC 8) $\quad {}_b(\alpha) \leftrightarrow \neg\Diamond_b(\neg\alpha)$.

According to DEON 2, the resulting formula $\neg\Diamond_b(\neg\alpha)$ is equivalent to $\neg E(\neg\alpha)$ which in turn entails the desired conclusion $E(\alpha)$ by way of the further theorem:

(DEON 13) $\quad \neg E(\neg\alpha) \rightarrow E(\alpha)$.

Note, incidentally, that in an earlier proof which was later deleted by Leibniz, the conclusion $\Diamond_b(\alpha)$ or $E(\alpha)$ had been obtained more directly by inferring $_b(\alpha)$ from the premise (α) and then making use of the following law which relativizes NEC 3 to the person *b*:

(NEC 9) $\quad {}_b(\alpha) \rightarrow \Diamond_b(\alpha)$

For, as Leibniz remarks: „Everything which is necessary for the virtuous man is anyway possible for the virtuous man as such, i.e. it is permitted"[31]. Similarly Leibniz proves DEON 12b as follows:

„Nothing which is impossible is obligatory, i.e. there is no obligation for impossibles.

For everything which is impossible is impossible for the virtuous man. Nothing which is impossible for the virtuous man is anyway possible for the virtuous man as such. What is not possible for the virtuous man as such is not necessary for the virtuous man as such, i.e. it is not obligatory".[32]

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30 Cf. **A** VI, 4, 2759/60: „Omne necessarium est licitum, seu necessitas non habet legem.Nam omne necessarium est necessarium viro bono. Quod est necessarium viro bono, ejus oppositum est impossibile viro bono. Quod impossibile viro bono utcunque non est possibile viro bono qua tali seu licitum. Ergo necessarii oppositum non est licitum. Cujus autem oppositum non est licitum, id ipsum est licitum."

31 Cf. **A** VI, 4, 2759: „Omne necessarium viro bono utcunque est possibile viro bono qua tali; hoc est licitum".

32 Cf. **A** VI, 4, 2759: „Nullum impossibile est debitum, seu impossibilium nulla est obligatio. Nam omne impossibile est impossibile viro bono. Nullum impossibile viro bono utcunque est possibile viro bono qua tali. Quod non est possibile viro bono qua tali non est necessarium viro bono qua tali, seu non est debitum."

Here again by means of the „bridge principle" NEC 7, $\neg\Diamond_b(\alpha)$ is first shown to follow from $(\neg\alpha)$ or $\neg\Diamond(\alpha)$; second, NEC 9 transformed by contraposition into $\neg\Diamond_b(\alpha) \rightarrow \neg\ _b(\alpha)$ is used to derive $\neg\ _b(\alpha)$ which, thirdly, according to DEON 1, gives the desired conclusion $\neg O(\alpha)$.

6 Literature

Editions of Leibniz's works

A Akademie-Ausgabe, i.e.: German Academy of Science, ed., G. W. Leibniz, *Sämtliche Schriften und Briefe*, esp. SeriesVI *Philosophische Schriften*, Darmstadt 1930, Berlin 1962 ff.

C Louis COUTURAT (ed.), *Opuscules et fragments inédits de Leibniz*, Paris 1903, reprint Hildesheim (Olms) 1960.

GI *Generales Inquisitiones de Analysi Notionum et Veritatum – Allgemeine Untersuchungen über die Analyse der Begriffe und Wahrheiten*, ed. by F. SCHUPP, Hamburg (Meiner), 1982.

GP C. I. GERHARDT (ed.), *Die philosophischen Schriften von G. W. Leibniz*, 7 volumes Berlin/Halle 1849-63, reprint Hildesheim (Olms) 1962.

Grua G. GRUA (ed.), *G. W. Leibniz – Textes inédits*, Paris 1948.

P G. W. Leibniz, *Logical Papers*, ed. and translated by G. H. R. PARKINSON, Oxford (Clarendon Press) 1966.

Other literature:

ADAMS, Robert M. [1982]: „Leibniz's Theories of Contigency", in M. Hooker (ed.): *Leibniz: Critical and Interpretive Essays*, Minneapolis (University of Minnesota Press), 243-83.

ANDERSON, Alan Ross [1958]: „A Reduction of Deontic Logic to Alethic Modal Logic", in *Mind* LXVII, 100-103.

LENZEN, Wolfgang [1980]: *Glauben, Wissen und Wahrscheinlichkeit*, Wien (Springer).

LENZEN, Wolfgang [1983]: „Zur extensionalen und „intensionalen“ Interpretation der Leibnizschen Logik“, in *Studia Leibnitiana* 15, 129-148.

LENZEN, Wolfgang [1984a]: „‘Unbestimmte Begriffe’ bei Leibniz“, in *Studia Leibnitiana* 16, 1-26.

LENZEN, Wolfgang [1984b]: „Leibniz und die Boolesche Algebra“, in *Studia Leibnitiana* 16, 187-203.

LENZEN, Wolfgang [1986]: „‘Non est’ non est ‘est non’ - Zu Leibnizens Theorie der Negation“, in *Studia Leibnitiana* 18, 1-37.

LENZEN, Wolfgang [1987]: „Leibniz’s Calculus of Strict Implication“, in J. Srzednicki (ed.) *Initiatives in Logic (Reason and Argument* 1), Dordrecht, 1-35.

LENZEN, Wolfgang [1990]: *Das System der Leibnischen Logik*, Berlin (de Gruyter).

LEWIS, Clarence I. & Cooper H. LANGFORD [1932]: *Symbolic Logic*, New York, ^{2}1959 (Dover Publications).

POSER, Hans [1969]: *Zur Theorie der Modalbegriffe bei G. W. Leibniz*, Wiesbaden (Steiner).

QUINE, Willard v. O. [1953]: *From a Logical Point of View*, New York (Harper & Row).

RESCHER, Nicholas [1954]: „Leibniz’s interpretation of his logical calculus“, in *Journal of Symbolic Logic* 19, 1-13.

SCHEPERS, Heinrich [1975]: „Leibniz’ Disputation ‚De Conditionibus’: Ansätze zu einer juristischen Aussagenlogik“, in *Akten des II. Internationalen Leibniz-Kongresses*, Bd. V, 1-17.

SWOYER, Chris [1995]: „Leibniz on Intension and Extension“, in *Noûs* 29, 96-114.