KNOWLEDGE, BELIEF, EXISTENCE, AND QUANTIFIERS – A NOTE ON HINTIKKA*

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1

Jaakko Hintikka's pioneering book *Knowledge and Belief*[1] has prompted a very extensive discussion over the past 15 years during the course of which practically no proposed principle for a logic of knowledge and belief has been spared from critical objections.[2] The least objectionable epistemic principle, and in fact the one least objected to, is the common sense truism that false statements cannot be known (to be true):

P1: If some person *a* knows that *p*, then *p* must in fact be true.

Adopting Hintikka's symbolic notation, this fundamental principle can be represented by the formula

(1) $K_a p \supset p$.

A somewhat refined, and equally plausible, version of **P1** might be formulated as follows:

P2: If there is an individual, *x*, such that some person *a* knows that *p* is true of *x*, then there must in fact be an individual of whom *p* is true.

Although the meaning and use of quantifiers within epistemic contexts will not be explained until section 3 below, we may now symbolize **P2** anticipatorily by

(2) $(\exists x)K_a p \supset (\exists x)p$.

It is easily seen that (2) is entailed by (1) in conjunction with the following third principle:

P3: If there is an individual, *x*, such that some person *a* knows that *p* is true of *x*, then *a* knows *a fortiori* that there is an individual of whom *p* is true.

Formally:

(3) $(\exists x)K_a p \supset K_a(\exists x)p$.

The analogues of (3) for both the alethic and the deontic modalities usually are accepted as indispensable principles for quantified modal logic. In the same manner we must presumably accept not only (3) itself but also the doxastic counterpart thereof:

P4: If there is an individual, *x*, such that person *a believes* that *p* is true of *x*, then *a* believes *a fortiori* that there is an individual of whom *p* is true.

Formally:

(4) $(\exists x)B_a p \supset B_a(\exists x)$p.

For instance, if there is an individual, *x*, who is believed by Tom to be the assassin of J. F. Kennedy, i.e. if Tom believes that he knows Kennedy's assassin (or – as Hintikka prefers to say – if Tom has an opinion about who Kennedy's assassin is), then Tom

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1 J. Hintikka, *Knowledge and Belief*, Ithaca, N.Y.: Cornell University Press, 1962.

2 The literature on this field is reviewed in an article of mine on "Recent Work in Epistemic Logic" (*Acta Philosophica Fennica* XXX (1978), Issue 1).

believes *ipso facto* that there is some such assassin, i.e. Tom believes that the then President of the United States actually was assassinated. Presumably this example suffices to illustrate the plausibility of **P4**.

It would seem to be intuitively clear that Hintikka should have defended the above principles as he did in *Knowledge and Belief*[3]; but, surprisingly, in later works he came to consider them as "unacceptable". The purpose of the present note is to show that his grounds for rejecting **P2-P4** are not conclusive and that these principles may, accordingly, be retained as cornerstones of quantified epistemic logic.

2

To fully understand and appreciate the difficulties to which **P2-P4** seem to give rise, let us first examine Hintikka's treatment of *existence.* This turns out to be a crucial point for the problems under discussion. In his various logical writings, Hintikka expressly allows the use of "empty" singular terms, i.e. of individual terms (both names and descriptions) that do not refer to actually existing entities. This liberality enables him to deal logically with sentences such as the notorious 'Pegasus is a flying horse', which may then be expressed symbolically by formulae of the type

(5) $F(b)$.

With respect to quantifiers, however, Hintikka retains the "ordinary" or "standard" interpretation according to which, for instance, "$(\forall x)p$ is understood to mean 'of each *actually existing* individual (call it x) it is true that p'."[4] Given this combined interpretation characteristic not only of Hintikka's system but also of various other systems of "free logic", the usual principles of existential generalization and universal instantiation will no longer be unrestrictedly valid. For instance, from (5) we cannot infer

(6) $(\exists x)F(x)$,

for despite Pegasus' being a flying horse, such a flying horse does not actually exist. Existential generalization and universal instantiation require an additional premise, viz., that the entity referred to by b actually exists. As was proved by Hintikka (and, almost simultaneously, by K. Lambert)[5], this existence assumption "will necessarily have the same logical powers as" the formula

(7) $(\exists x)(x=b)$.

This formula thus represents a definiens for the predicate of existence in terms of which statements of non-existence also can be formulated. The negation of (7), e.g., may be regarded as a proper symbolization of the statement 'the individual "referred" to by b (by 'Pegasus', in our example) does not actually exist', or, somewhat elliptically, 'b (Pegasus) does not exist'.

However, there are other statements about non-existents which cannot be expressed using this approach, although one might wish these to be expressible. Let us consider Greek mythology, for example! If we take the problem of "empty" singular terms seriously, i.e. if we take statements containing them to be meaningful, then we will have to admit, e.g., that 'Zeus has at least one daughter' is true once we admit that 'Pallas Athena is

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[3] Cf. Hintikka, o.c. (1962), pp. 160-2.
[4] J. Hintikka, "Existential Presuppositions and Their Elimination", in J. Hintikka, *Models for Modalities*, Dordrecht, Holland: D. Reidel Publishing Company, 1969, p. 26.
[5] Cf. Hintikka, o.c. (1969), pp. 30-1, and K. Lambert, "Free Logic and the Concept of Existence", *Notre Dame Journal of Formal Logic*, 8, 1967, p. 141.

a daughter of Zeus' is true. Now, whereas the latter statement may be rendered symbolically by

(8) $G(c,d)$,

the straightforward transcription of the former, viz.,

(9) $(\exists x)G(x,d)$,

evidently will not do, because no such goddess actually exists. Similarly, there is no appropriate symbolic counterpart for the statements 'Pallas Athena is not the only Greek goddess', 'Zeus has at least four spouses', etc.

Now, these shortcomings may not seem very serious as long as only predicate logic is concerned. Difficulties increase, however, if we return to epistemic logic and investigate more closely the consequences of the fundamental principles stated in section 1 above.

3

It is well known that, apart from existential presuppositions, further restrictions are needed in order to validate quantification into epistemic contexts. For instance, Tom's almost tautological knowing that the first man to climb Mt. Everest climbed that mountain, can be analyzed as having the logical form

(10) $K_a p(b)$.

But we do not want to infer that there hence is an individual, x, such that Tom knows x climbed Mt. Everest,

(11) $(\exists x)K_a p(x)$.

The former statement seems to be true as soon as Tom knows that someone first set foot on that mountain, whereas the truth of the latter statement seems to require in addition that Tom knows *of* someone, e.g. of Sir Hilary, that he climbed it. As has been advocated especially by Hintikka, the inference from (10) to (11) should be considered legitimate if and only if b refers to one and the same individual in all of a's *epistemic alternatives*, i.e. in all possible worlds which are compatible with everything a knows.

In his contribution to the *Noûs*-symposium on epistemic logic[6], Hintikka proved that the latter condition is equivalent to the validity of

(12) $(\exists x)K_a(x=b)$,

which is to be read as 'a knows who b is'. Thus, in the preceding example, the inference in question will be valid if and only if Tom knows who the first man was who climbed Mt. Everest. Similarly, it can be shown that the necessary and sufficient condition for quantifying into doxastic contexts may be represented symbolically by the formula

(13) $(\exists x)B_a(x=b)$,

which Hintikka reads as 'a has an opinion about who b is'.

Now it is easy to see where the difficulties hinted at in section 1 arise from. If we substitute, for instance, '$x=b$' for 'p' in our former principles (2), (3), and (4), we obtain

(14) $(\exists x)K_a(x=b) \supset (\exists x)(x=b)$,
(15) $(\exists x)K_a(x=b) \supset K_a(\exists x)(x=b)$,

and

(16) $(\exists x)B_a(x=b) \supset B_a(\exists x)(x=b)$.

[6] J. Hintikka, "Individuals, Possible Worlds, and Epistemic Logic", *Noûs* 1, 1967, pp. 33-62; the proof is given on pp. 35-8.

Given the intended reading of (7), (12), and (13), the sentences (15) and (16) thus amount to saying that you can know or have an opinion about who *b* is only if you know or believe that *b* actually exists. And (14) states that you cannot know who *b* is unless *b* actually exists. What seemed to be surprising at first sight now becomes plain: these principles, at least in their informal version, really are unacceptable. As Hintikka himself came to admit, (15), or better its ordinary language counterpart, must be rejected since "there seems to be a perfectly good sense of knowing who a certain person is which does not commit one to holding that the person in question is known to exist."[7] And with respect to (14), P. Weingartner argued that an equally good sense of knowing who *b* is must be admitted even if *b* in fact does not exist. For, according to Weingartner, the question 'Do you know who Polyphemus was?' may be answered quite correctly by saying 'He was the one-eyed giant in the *Odyssey*'.[8] Finally, as regards (16), or more precisely (4) from which (16) is derived, Hintikka pointed out that its acceptance "would make the following sentence self-sustaining (logically true)

(*) $$(\exists x)B_a{\sim}(\exists y)(x{=}y) \supset B_a(\exists x){\sim}(\exists y)(x{=}y),$$

i.e. it would make the antecedent of (*) contradictory, which certainly is nonsensical."[9] Clearly, one may have an opinion about some actually existing individual, *x*, even if one (falsely) believes that *x* does not actually exist.

Hintikka countered all these difficulties by simply dropping (2)-(4), and thus **P2**-**P***4* as well. This, however, is very unsatisfactory. On the one hand, given the "ordinary" interpretation of the existential quantifier, (2) ought after all to come out as true, just as the informal principle **P2** remains beyond reasonable doubt. And on the other hand, we surely would want any system of quantified epistemic/doxastic logic to contain a symbolic counterpart of the two other principles **P3** and **P4**. It ought by now to be clear that the difficulties revealed by the foregoing objections do not concern the adequacy of **P2**, **P3** or **P4** as such, but rather challenge the appropriateness of (12) and (13) as formal representatives of the prerequisites for quantifying into epistemic and doxastic contexts.[10] If knowing or having an opinion about who *b* is must not be taken to presuppose that *b* actually exists or is known or believed to exist, then evidently (12) and (13) should be replaced by formulae which carry no such existential presuppositions. However, such formulae are not available within Hintikka's approach.

4

Fortunately, concerning the issue of existence C. Lejewski and others in the mid-50's developed a quite different approach which is becoming increasingly popular. More recently, it has been advocated by B. C. van Fraassen, K. Lambert, and D. Scott, to name only a few prominent representatives.[11] The basic idea of this approach is to include the domain of

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[7] J. Hintikka, "'Knowing Oneself' and Other Problems in Epistemic Logic", *Theoria* 32, 1966, p. 4. This article was a reply to H.-N. Castañeda's review of *Knowledge and Belief* in *The Journal of Symbolic Logic* 29, 1964, pp. 132-4, where the principles in question were already critically examined. Cf. also D. Follesdal, "Knowledge, Identiy, and Existence", *Theoria* 33, 1967, pp. 1-27, and Hintikka's comments, *ibid*., pp. 138-47. Castañeda repeated his criticisms in "On the Logic of Self-Knowledge", *Noûs* 1, 1967, pp. 9-21.

[8] Cf. P. Weingartner, "Sind das Cogito und ähnliche Existenzsätze zum Teil analytisch? " in Weingartner (*ed*.), *Deskription, Analytizität und Existenz*, Salzburg and Munich, 1966, pp. 310-1.

[9] J. Hintikka, o.c. (1967), p. 39, fn. 8.

[10] Cf. R. C. Sleigh, "On a Proposed System of Epistemic Logic", *Noûs* 2, 1968, p. 394, where Hintikka's rejection of (2) is said "to obscure the interpretation of the existential quantifier and/or the basic epistemic operator". However, Sleigh does not propose an alternative solution as we will do in what follows.

[11] 11. C. Lejewski's "Logic and Existence", *The British Journal for the Philosophy of Science* 5, 1954, pp. 104-19, seems to be the first presentation of a logic of existence as sketched below. For the more recent defense of the possible object semantics, cf., e.g., K. Lambert and B. C. van Fraassen, "Meaning Relations, Possible

actually existing entities within the domain of all possible individuals and then let the quantifiers range over the entire set. The concept of a possible, non-existent object has been subjected to severe criticism, notably by W. V. Quine.[12] However, it is not necessary to discuss the general ontological problems involved by that notion, especially since Hintikka's treatment of existence within and without epistemic contexts, which is at issue here, explicitly deals with "nonexisting individuals".[13] In this company, therefore, I rather take it for granted that quantifying over possibles can be given a perfectly good sense.

If we denote the possible object quantifiers by '$\mathrm{V}x$' and 'Λx', then Hintikka's *restricted* quantifiers[14] can easily be defined by

(17) $(\exists x)p := \mathrm{V}x(\mathcal{E}(x) \ \& \ p)$

(18) $(\forall x)p := \Lambda x(\mathcal{E}(x) \supset p)$.[15]

Here $\mathcal{E}$ is a one place predicate constant intended to express actual existence. Since the principles of "existential" generalization and universal instantiation are taken to hold for $\mathrm{V}x$ and Λx without restriction, this predicate of existence coincides with Hintikka's, i.e. we have

(19) $(\exists x)(x=b) \equiv \mathcal{E}(b)$.

Now, the use of the possible object quantifiers seems to have great advantages. First, it permits formalizing those statements about non-existents which, as was indicated in section 2 above, cannot be expressed upon the "standard" approach. Second, and more important it provides a symbolization of the prerequisites for quantifying into epistemic and doxastic contexts more adequate than (12) and (13). Semantically speaking, the conditions '*a* knows who *b* is' and '*a* has an opinion about who *b* is' are tantamount to demanding that the referent of *b* remains constant in all of *a*' s epistemic or doxastic alternatives. But Hintikka explicitly admits the use of non-referring terms (which, upon our approach, "refer" to possibles). Furthermore he explicitly acknowledges certain knowings and believings about their "referents" as sufficient for quantification. Thus, these conditions naturally must be taken to mean, more precisely, that *b* "refers" to one and the same *possible* individual in all relevant possible worlds. (12) and (13) accordingly must be replaced by

(12′) $\mathrm{V}xK_a(x=b)$

and

(13′) $\mathrm{V}xB_a(x=b)$.

In terms of this "weak sense" of knowing or having an opinion about who someone is, the fundamental principles **P2**-**P4** can now be formulated more adequately by the sentences

(2′) $\mathrm{V}xK_ap \supset \mathrm{V}xp$

(3′) $\mathrm{V}xK_ap \supset K_a\mathrm{V}xp$

Objects, and Possible Worlds" and D. Scott's "Advice on Modal Logic", both in K. Lambert (ed.), *Philosophical Problems in Logic*, Dordrecht, Holland: D. Reidel Publishing Company, 1970.

[12] W. V. Quine, "On What There Is", *Review of Metaphysics* 2, 1948, pp. 21-38. Cf., however, N. Rescher's attempted invalidation of Quine's criticism in "The Logic of Existence", *The Philosophical Review* 68, 1959, especially pp. 172-4.

[13] Cf. Hintikka, o.c. (1969), p. 28.

[14] The name 'restricted' is somewhat ambiguous since quantification into epistemic contexts involves a twofold restriction of the domain of discourse to individuals which are both actually existing and known by the respective person *a*. 'Restricted quantification' is usually taken to refer to the latter restriction; here, however, the former is meant.

[15] These definitions seem to be due to B.C. Fraassen. Cf., e.g., his "Meaning Relations among Predicates", *Noûs* 1, 1967, pp. 161-179. This was pointed out to me by Prof. F. von Kutschera to whom I am grateful for having roused my interest in intensional logic

and

(4′) $\mathrm{V}xB_a p \supset B_a \mathrm{V}xp$.

In the remainder of this note, I will try to make these principles intelligible and plausible.

5

First of all, recalling definition (17), it can easily be proven that the former principle (2) is valid. In agreement with what we intuitively expected, the antecedent of (2), viz.

(20) $\mathrm{V}x(\mathcal{E}(x) \ \& \ K_a p)$,

in virtue of (2′) directly entails the consequent,

(21) $\mathrm{V}x(\mathcal{E}(x) \ \& \ p)$.

However, (3′) and (4′) do not analogously entail (3) and (4), which may be illustrated as follows:

Suppose a friend of mine, *a*, knows that the next book I am going to write, *b*, will be about epistemic logic. Hence we have

(22) $K_a p(b)$,

where the open sentence *p(x)* stands for ‘*x* is a book on epistemic logic’. Suppose further that we have discussed my project so existensively that *a* knows (in a sense comparable to knowing *who b* is) *what b* is. Since *b* does not exist yet either in form of a completed manuscript or even as a proper book,

(23) $\sim \mathcal{E}(b)$,

a’s knowing what *b* is must not be expressed by Hintikka’s (12) but by our modified (12′). Also, the fact entailed by (22) plus (12′) that there is an “object” *x* such that *a* knows *x* is a book on epistemic logic, must not be symbolized by

(24) $(\exists x)K_a p$,

but by

(25) $\mathrm{V}xK_a p$.

Suppose finally the day has arrived when the book is eventually published, i.e. *b* has come into actual existence

(26) $\mathcal{E}(b)$;

however, being as yet uninformed about the publication, *a* does not know this:

(27) $\sim K_a(\exists \mathrm{x})(x=b)$.

Hence he also does not know that a book on epistemic logic (now) exist:

(28) $\tilde{\ } \mathrm{K}_\mathrm{a}(\exists x)p$.

We may, of course, assume for the sake of argument that *a* never heard about *Knowledge and Belief* or any other book on epistemic logic already published.

Now, *a*’s weakly knowing what *b* is, (12′), plus *b*’s actual existence (though unknown to *a*), (26), certainly suffice to infer *a*’s knowing what *b* is in the stronger sense of Hintikka ((12)). But in virtue of (3′) the sentence (12) does not give rise to the unwanted conclusion

(29) $K_a(\exists x)(x=b)$,

as would have been the case had Hintikka’s principle (3) been adopted.

Similarly, (25) plus (26) entail the existence of an object, x, known by a to be a book on epistemic logic, (24). But again (3′) does not imply that a hence knows there exists a book on epistemic logic,

(30) $K_a(\exists x)p$,

as would have been the case had (3) been used.

Let me illustrate the plausibility of the corresponding implications and non-implications in the case of belief by the following, concluding example. Let $F(x)$ be the predicate of being a Greek goddess, and let a, b denote Tom and Hera, respectively. Let us further assume that Tom has an opinion about who Hera is and that Tom believes Hera to be a Greek goddess. We then have

(31) $\mathrm{V}xB_a(x=b)\ \&\ B_aF(b)$.

From (31) we want and, indeed, are allowed to infer that there is an individual – whether it actually exists or not – which is believed by Tom to be a Greek goddess,

(32) $\mathrm{V}xB_a\mathrm{F}(x)$.

Thus, in view of (4′), Tom has to believe *a fortiori* that there are – actually existing or not actually existing – Greek goddesses,

(33) $B_a\mathrm{V}xF(x)$.

If Tom ought to believe, moreover, that Hera actually exists,

(34) $B_a\mathcal{E}(b)$,

we might further conclude that Tom believes there actually exists at least one Greek goddess,

(35) $B_a(\exists x)\mathrm{F}(x)$.

This follows even though the individual which is believed by Tom to be a Greek goddess does not actually exist so that

(36) $(\exists x)B_aF(x)$

is false.

In a world other than ours Hera might have actually existed,

(37) $\mathcal{E}(b)$.

Given this alternative, we find that (31) in conjunction with (37) would have implied (36) without, however, thereby guaranteeing the truth of (35). Here again (35) would have been warranted only if (34) were true.

I consider these implications to be the very ones we want to hold. The possible-object-approach to existence thus enables us to formulate principles which evidently have to be incorporated into any adequate system of epistemic/doxastic logic. This approach furthermore helps to account for the failure of the corresponding principles as formulated upon Hintikka’s approach.