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Knowledge, Belief, and Subjective Probability - Outlines of a Unified System of Epistemic/Doxastic Logic

0 Foreword

The aims of this paper are (i) to summarize the *semantics* of (the propositional part of) a unified epistemic/doxastic logic as it has been developed at greater length in Lenzen [1980] and (ii) to use some of these principles for the development of a semi-formal *pragmatics* of epistemic sentences. While a semantic investigation of epistemic attitudes has to elaborate the truth-conditions for, and the analytically true relations between, the fundamental notions of belief, knowledge, and conviction, a pragmatic investigation instead has to analyse the specific conditions of rational utterance or utterability of epistemic sentences. Some people might think that both tasks coincide. According to Wittgenstein, e.g., the *meaning* of a word or a phrase is nothing else but its *use* (say, within a certain community of speakers). Therefore the pragmatic conditions of utterance of words or sentences are assumed to determine the meaning of the corresponding expressions. One point I wish to make here, however, is that one may elaborate the meaning of epistemic expressions in a way that is largely independent of - and, indeed, even partly incompatible with - the pragmatic conditions of utterability. Furthermore, the crucial differences between the pragmatics and the semantics of epistemic expressions can satisfactorily be explained by means of some general principles of *communication*. In the first three sections of this paper the logic (or semantics) of the epistemic attitudes *belief*, *knowledge*, and *conviction* will be sketched. In the fourth section the basic idea of a general pragmatics will be developed which can then be applied to epistemic utterances in particular.

1 The Logic of Conviction

Let ‘$C(a,p)$’ abbreviate the fact that person *a* is firmly convinced that *p*, i.e. that *a* considers the proposition *p* (or, equivalently, the state of affairs expressed by that proposition) as absolutely certain; in other words, *p* has maximal likelihood or probability for *a*. Using ‘Prob’ as a symbol for subjective probability functions, this idea can be formalized by the requirement:

(PROB-C) $C(a,p) \leftrightarrow \text{Prob}(a,p)=1.$

Within the framework of standard possible-worlds semantics $<I,R,V>$, $C(a,p)$ would have to be interpreted by the following condition:

(POSS-C) $V(i,C(a,p))=\text{t} \leftrightarrow \forall j(iRj \rightarrow V(j,p)=\text{t}).$

Here I is a non-empty set of (indices of) possible worlds; R is a binary relation on I such that iRj holds iff, in world i, a considers world j as possible; and V is a valuation-function assigning to each proposition p relative to each world i a truth-value $V(i,p)\in\{\text{t},\text{f}\}$. Thus $C(a,p)$ is true (in world $i\in I$) iff p itself is true in every possible world j which is considered by a as possible (relative to i).

The probabilistic “definition” **POSS-C** together with some elementary theorems of the theory of subjective probability immediately entails the validity of the subsequent laws of conjunction and non-contradiction. If *a* is convinced both of *p* and of *q*, then *a* must also be convinced that *p* and *q*:

(C1) $C(a,p) \wedge C(a,q) \rightarrow C(a,p\wedge q).$

For if both Prob(a,p) and Prob(a,q) are equal to 1, then it follows that Prob($a,p\wedge q$)=1, too. Furthermore, if a is convinced that p (is true), a cannot be convinced that $\neg p$, i.e. that p is false:

(C2) $C(a,p) \rightarrow \neg C(a,\neg p)$.

For if Prob(a,p)=1, then Prob($a,\neg p$)=0, and hence Prob($a,\neg p$)$\neq$1. Just like the alethic modal operators of possibility, $\Diamond$, and necessity, $\Box$, are linked by the relation $\Diamond p \leftrightarrow \neg\Box\neg p$, so also the doxastic modalities of thinking p to be possible - formally: $P(a,p)$ - and of being convinced that p, $C(a,p)$, satisfy the relation

(Def. P) $P(a,p) \leftrightarrow \neg C(a,\neg p)$.

Thus, from the probabilistic point of view, $P(a,p)$ holds iff a assigns to the proposition p (or to the event expressed by that proposition) a likelihood greater than 0:

(PROB-P) $V(P(a,p))$=t $\leftrightarrow$ Prob(a,p)>0.

Within the framework of possible-worlds semantics, one obtains the following condition:

(POSS-P) $V(i,P(a,p))$=t $\leftrightarrow \exists j(iRj \wedge V(j,p)$=t),

according to which $P(a,p)$ is true in world i iff there is at least one possible world j - i.e. a world j accessible from i - in which p is true.

In view of **Def P**, the former principle of consistency, **C2**, can be paraphrased by saying that whenever a is firmly convinced that p, a will a fortiori consider p as possible. However, considering p as possible does not conversely entail being convinced that p. In general there will be many propositions p such that a considers both p and $\neg p$ as possible. Such a situation, where $P(a,p) \wedge P(a,\neg p)$, makes clear that unlike the operator C, P does not satisfy a principle of conjunction analogous to **C1**. However, the converse entailment

(C3) $P(a,p\wedge q) \rightarrow P(a,p) \wedge P(a,q)$

and its counterpart

(C4) $C(a,p\wedge q) \rightarrow C(a,p) \wedge C(a,q)$

clearly are valid, because the probabilities of the single propositions p or q always are at least as great as the probability of the conjunction ($p\wedge q$). Similarly, since the probability of a disjunction ($p\vee q$) is always at least as great as the probabilities of the single disjuncts p and q, it follows that both operators C and P satisfy the following principles of disjunction:

(C5) $C(a,p) \vee C(a,q) \rightarrow C(a,p\vee q)$

(C6) $P(a,p) \vee P(a,q) \rightarrow P(a,p\vee q)$.

Now the probabilistic "proofs" of such principles are not without problems. Since its early foundations by De Finetti [1964], the theory of subjective probability has always been formulated in terms of *events*, while in the framework of philosophical logic, attitudes like $C(a,p)$ are traditionally formulated in terms of *sentences*. So if one wants to apply the laws of the theory of subjective probability to the fields of cognitive attitudes, one has to presuppose (i) that for every event X there corresponds exactly one proposition p, and (ii) that the cognitive attitudes really are "prepositional" attitudes in the sense that their truth is independent of the specific linguistic representation of the event X. That is, whenever two sentences p and q are logically equivalent and thus describe one and the same event X, then $C(a,p)$ holds iff $C(a,q)$ holds as well. This requirement can be formalized by the following rule:

(C7) $p \leftrightarrow q \vdash C(a,p) \leftrightarrow C(a,q)$.

This principle further entails that everybody must be convinced of everything that logically follows from his own convictions:

(C8) $p \to q \ \vdash\ C(a,p) \to C(a,q)$.

For if p logically implies q, then p is logically equivalent to $p \wedge q$; thus $C(a,p)$ entails $C(a,p \wedge q)$ (by **C7**) which in turn entails $C(a,q)$ by **C4**.

There has been a long discussion (still going on in the literature) whether and to which extent the cognitive attitudes of real subjects actually are *deductively closed*. In view of man's almost unlimited fallibility in matters of logic, some authors have come to argue that **C8** should be restricted to very elementary instances like **C4** or **C5** or to some other so-called 'surface tautologies'[1]. Which option one favours will strongly depend on the methodological role that one wants to assign to epistemic logic. If epistemic logic is conceived of as a *descriptive system* of people's factual beliefs, then not even the validity of elementary principles like **C4** seems warranted. If, however, epistemic logic is viewed as a *normative* system of *rational* belief, then even the strong condition of full deductive closure, **C8**, appears perfectly acceptable. Incidentally, if one presupposes that everybody has at least one conviction - an assumption which is logically guaranteed by some of the subsequent iteration-principles[2] - **C8** entails the further rule

(C9) $p \ \vdash\ C(a,p)$,

according to which everybody is convinced of every tautological proposition (or state of affairs) p.

To round off our exposition of the logic of conviction, let us consider some laws for *iterated* epistemic attitudes. According to the thesis of the "privileged access" to our own mental states, whenever some person a is convinced of p, a knows that she has this conviction. Similarly, if a is not convinced that p, i.e. if she considers p as possible, then again she knows that she considers p as possible:

(E1) $C(a,p) \ \to\ K(a,C(a,p))$

(E2) $\neg C(a,p) \ \to\ K(a,\neg C(a,p))$.

Here '$K(a,q)$' abbreviates the fact that a *knows* that q. Now, clearly a knows that q only if in particular a is convinced that q:

(E3) $K(a,p) \ \to\ C(a,p)$.

Hence **E1** and **E2** immediately entail the following purely doxastic iteration-principles:

(C10) $C(a,p) \ \to\ C(a,C(a,p))$

(C11) $\neg C(a,p) \ \to\ C(a,\neg C(a,p))$.

It is easy to verify that the implications **C10** and **C11** may be strengthened into equivalences. Hence iterated doxastic operators (or "modalities") are always reducible to simple expressions of the type $C(a,p)$ or $\neg C(a,q)$, where p and q contain no further doxastic expressions. As was proven in theorem **MT 15** of Lenzen [1980], iterated doxastic *propositions* of arbitrary complexity can be reduced to simple, non-iterated propositions. Technically speaking, the logic of conviction turns out to be structurally isomorphic to the "deontic" calculus **DE4** of [Lemmon 1977] which differs from the better-known alethic calculus **S5** only in that it does not contain the truth-axiom $\Box p \to p$. Given the intended doxastic interpretation of necessity as

[1] Cf., e.g., Hintikka [1970].

[2] Clearly, since $C(a,p) \vee \neg C(a,p)$ holds tautologically, **C10** and **C11** entail that $C(a,C(a,p)) \vee C(a,\neg C(a,p))$ is epistemic-logically true. So either way there exists a q such that $C(a,q)$.

subjective necessity or certainty, the failure of $C(a,p) \rightarrow p$ comes as no surprise. After all, humans are not infallible; therefore someone's conviction that p - however firm it may be - can never logically guarantee that p is in fact the case.

2. The Logic of Knowledge

Although, then, a's conviction that p is logically compatible with p's actually being false, it is a truism since Plato's early epistemological investigations in the *Theaitetos* that a cannot *know* that p unless p is in fact true. This first ("objective") condition of knowledge can be formalized as:

(K1) $K(a,p) \rightarrow p.$

Another (subjective) condition of knowledge has already been stated in the preceding section. **E3** says that person a cannot know that p unless she is convinced that p. This is a refinement of Plato's insight that knowledge requires belief - *viz*., belief of the strongest form possible. Plato had discussed yet a third condition of knowledge which is somewhat harder to grasp. In order really to constitute an item of *knowledge*, a's true belief must be "justified" or "well-founded". One might think of explicating this requirement by postulating the existence of certain proposition $q_1, ..., q_n$ which justify a's belief that p by logically entailing p. But which epistemological status should be accorded to these justifying propositions? If it were only required that the q_i must all be true and that a is convinced of their truth, then the third condition of knowledge would become redundant and each true belief would by itself be justified.[3] On the other hand one cannot require that the q_i are *known* by a to be true, because then Plato's definition of knowledge as justified true belief would become circular.

Therefore one will either treat 'knowledge' as a primitive, indefinable notion which is characterized only partially by the necessary conditions **K1** and **E3**. Or one takes the conjunction of these two conditions as already *sufficient* for a's knowing that p - an option favoured, e.g., by Kutschera [1976]. Let us refer to this simple concept of knowledge as 'knowledge*' or 'K^*'. If one thus defines:

(Def. K*) $K^*(a,p) \leftrightarrow C(a,p) \wedge p,$

the logic of knowledge* can entirely be derived from the logic of conviction. The former conjunction-principle **C1** immediately entails the corresponding principle $K^*(a,p) \wedge K^*(a,q) \rightarrow K^*(a,p\wedge q)$, *but* conjunctivity seems to be valid also for the more demanding concept of knowledge along Plato's lines:

(K2) $K(a,p) \wedge K(a,q) \rightarrow K(a,p\wedge q).$

For if one assumes that a's single beliefs that p and that q are justified, then a's belief that $(p\wedge q)$ would be justified as well. Furthermore, the rules of deductive closure of conviction, **C7 - C9**, directly entail corresponding rules for K^* which, again, seem to be valid also for the more ambitious concept K:

(K3) $p \leftrightarrow q \vdash K(a,p) \leftrightarrow K(a,q)$

(K4) $p \rightarrow q \vdash K(a,p) \rightarrow K(a,q)$

(K5) $p \vdash K(a,p).$

Provided that epistemic logic is taken as a *normative* theory of *rational* (or "implicit") attitudes, these rules are just as acceptable as their doxastic counterparts.

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[3] Clearly, if $C(a,p) \wedge p$, then there exist some $q_1,...,q_n$ such that the q_i are true and $C(a,q_i)$ and $\{q_1, ...,q_n\}$ logically entail p, *viz*., $q_1 = \quad = q_n = p$!

It is easy to verify that **Def. K*** together with **E1** or **C10** entails the iteration law $K^*(a,p) \rightarrow K^*(a,K^*(a,p))$. The corresponding so-called "KK-thesis" (formulated for the general concept of knowledge, K) says that whenever a knows that p, a knows that he (or she) knows that p:

(K6) $$K(a,p) \rightarrow K(a,K(a,p)).$$

In the literature surveyed in Lenzen [1978], several „counter-examples“ have been constructed to show that someone may know something without knowing that he knows. For instance, assume that during an examination candidate *a* answers the question in which year Leibniz was born by saying: 'In 1646'. The fact that *a gave* the correct answer usually is taken as sufficient to conclude that *a knew* the correct answer. But *a* may not have known at all that he knew it; in fact he may have thought he was just guessing. Such examples typically play on the ambiguity of the English verb 'to know' which has the meaning both of the German 'wissen' and of 'kennen'. In the former case, 'to know' is followed by a that-clause and then expresses a *propositional attitude*; while in the latter case, 'to know' is part of a direct object construction ('to know the answer'; 'to know the way to the railway station'; 'to know London'; etc) and then expresses no such attitude. Therefore the above "counter-example" fails to refute **K6** since *a*'s "knowing" the correct answer, or his knowing the year in which Leibniz was born, does not represent a propositional attitude as would be required by **K6**. According to the premises of the story, *a* did *not* know *that* Leibniz was born in 1646 because she was not at all certain of the date. If a person really *knows* that Leibniz was born in 1646 (i.e., by **E3**, if *a* in particular is *convinced* that Leibniz was born in 1646), then *a* can never *believe that she does not know* that Leibniz was born in 1646.

The argument contained in the preceding passage contains an application of another important principle which establishes an epistemic logical connection between all the three basic notions of knowledge, belief, and conviction. In its general form, it would have to be put as follows: Whenever person *a* is *convinced* that *p*, she will *believe* that she *knows* that *p*. With '$B(a,p)$' abbreviating '*a* believes that *p*', this principle takes the symbolic form:

(E4) $$C(a,p) \rightarrow B(a,K(a,p)).$$

In view of certain iteration laws discussed earlier in this paper, **E4** can be strengthened into the statement that when *a* is convinced that *p*, she must be convinced that she knows that *p*.

(E5) $$C(a,p) \rightarrow C(a,K(a,p)).$$

As was kindly pointed out to me by Vincent Fredricks, Lamarre/Shoham [1994] and other recent authors refer to **E5** as "Moore's Principle" because the basic idea that being certain entails being certain that one knows is thought to have first been put forward in Moore [1959][4]. Incidentally, the implications **E4** and **E5** might further be strengthened into equivalences, and because of **C10** also the following law becomes provable:

(E6) $$C(a,C(a,p)) \leftrightarrow C(a,K(a,p)).$$

E6 shows that knowledge and conviction are *subjectively* indiscernible in the sense that person *a* cannot tell apart whether she is "only" convinced that *p* or whether she really knows that *p*. This observation does not remove, however, the *objective* difference between *a*'s being convinced that *p* and *a*'s knowing that *p*; only the latter but not the former attitude entails the

[4] A careful analysis of Moore's paper doesn't, however, fully confirm this assumption. Although "Certainty" contains a lot of interesting epistemological observations, Moore all too often appears to conflate the (semantic) truth-conditions for $C(a,p)$ and $K(a,p)$ on the one hand and the pragmatic conditions for the utterability of the corresponding assertions: 'I am certain that p' and 'I know that p' on the other hand. Similar comments apply to Wittgenstein's detailed commentary on Moore in [1969]. Cf. Lenzen [1980a].

truth of *p*. Therefore it is always (objectively) possible that *a* is convinced of something which as a matter of fact is not true; but person *a* herself can never think this to be possible.

Because of the objective possibility of $C(a,p) \wedge \neg p$, the *K*-analogue of the doxastic iteration principle **C11**, i.e. $\neg K(a,p) \rightarrow K(a,\neg K(a,p))$, fails to hold. From the assumption that *a does not know* that *p* one cannot infer that she *knows* that she does not know that *p*. For if *a mistakenly* believes that she knows that *p*, i.e. if $C(a,p) \wedge \neg p$, one has $\neg K(a,p)$ (because of **K1)** and yet *a* does not know of her mistake, because in view of **E4** *a* believes that she *does* know that *p*; hence she is far from believing (or even knowing) that she does *not* know that *p*.

Technically speaking the logic of knowledge is isomorphic to a modal calculus at least as strong as **S4** but weaker than **S5**. Now there is a very large - indeed, as shown in Fine [1974], an *infinite* - variety of modal systems between **S4** and **S5**. E.g., so-called system **S4.2** is characterized by an axiom which - with ‘necessity’ interpreted as ‘knowledge’ - takes the form:

(K7) $\neg K(a,\neg K(a,p)) \rightarrow K(a,\neg K(a,\neg K(a,p)))$.

Another calculus **S4.4** is axiomatized by (the $\Box$-counterpart of):

(K8) $p \wedge \neg K(a,\neg K(a,p)) \rightarrow K(a,p))$.

However, the meaning or validity of these principles is not at all evident because common sense says little or nothing about the epistemic counterpart of the alethic modality $\Diamond\Box p$, i.e. $\neg K(a,\neg K(a,p))$. Fortunately, the laws of epistemic logic developed earlier in this paper give us a clue how to understand this complex term. It can be proven that person *a* is *convinced* that *p* iff she does *not know that she does not know* that *p*:

(E7/K9) $\neg K(a,\neg K(a,p)) \leftrightarrow C(a,p)$.

One the one hand, $C(a,p)$ entails $C(a,K(a,p))$ (by **E5**) and a fortiori $\neg C(a,\neg K(a,p))$ (by **C2**) and thus also $\neg K(a,\neg K(a,p))$ (by **E3**); on the other hand $\neg C(a,p)$ implies $K(a,\neg C(a,p))$ (by **E2**) and hence also $K(a,\neg K(a,p))$ (by rule **K4** in conjunction with **E3**).

In view of **K9**, then, the **S4.2**-like principle **K7** amounts to saying that when person *a* is convinced that *p*, she knows that she is convinced that *p* - this is exactly the contents of our earlier principle **E1**. Similarly, **S4.4**-like principle **K8** states that when *p* is true and when *a* is convinced that *p*, then *a* already knows that *p*. This holds true only of the simple concept of knowledge as true conviction, but not of Plato’s more demanding conception of knowledge as justified true conviction. Summing up one may say that the logic of knowledge is isomorphic to an alethic modal system *at least as strong as* **S4.2** *and at most as strong as* **S4.4**.[5]

To conclude our discussion of the logic of knowledge, let it just be pointed out that a possible-worlds semantics for *K* can be given along the following lines:

(POSS-K) $V(i,K(a,p))=\mathrm{t} \leftrightarrow \forall j(iSj \rightarrow V(j,p)=\mathrm{t})$.

Here ‘*S*’ denotes an accessibility relation between worlds which obtain iff world *j* is compatible with (or possible according to) all that *a* knows in world *i*.

3 The Logic of (“weak”) Belief

The concept of *conviction*, $C(a,p)$, has been defined above to obtain only if person *a* is absolutely *certain* that *p*. The more general concept of *belief*, $B(a,p)$, will be satisfied by the much weaker requirement that person *a* considers *p* as *likely* or as *probable*, where the lower

[5] A discussion of further candidates for the logic of knowledge may be found in Lenzen [1979].

bound of (subjective) probability may reasonably be taken to be .5, i.e. person *a* will not believe that *p* unless she considers *p* as more likely than not:

(PROB-B) $B(a,p) \leftrightarrow \mathrm{Prob}(a,p)>1/2.$

The notion of “weak” belief also satisfies the principle of non-contradiction analogous to **C2**:

(B1) $B(a,p) \rightarrow \neg B(a,\neg p).$

Clearly, if *p* has a probability greater than 1/2, then $\neg p$ must have a probability less than 1/2. On the other hand, $B(a,p)$ does not satisfy the counterpart of conjunction principle **C1**, because even if two single propositions *p* and *q* both have a probability > .5, it may well happen that $\mathrm{Prob}(a,p\wedge q)$ is as small as the product $\mathrm{Prob}(a,p) \bullet \mathrm{Prob}(a,q)$ and hence < .5. For instance, let an urn contain two black balls and one white ball where one of the black balls is made of metal while the white ball and the other black ball is made of wood. Now if just one ball is drawn from the urn at random, the probability of *p* = ‘The ball is black’ equals 2/3 and is thus > 1/2; also the probability of *q* = ‘The ball is made of wood’ is 2/3 > 1/2. But the probability of the joint proposition $(p\wedge q)$ = ‘The ball is made of wood and is black’ only is 1/3.

It follows from the theory of probability that conjunctivity of belief is warranted only in the special case where one of the two propositions is *certain*:

(E8) $B(a,p) \wedge C(a,q) \rightarrow B(a,p\wedge q).$

Here certainty may be said to represent a special instance of belief in the sense of:

(E9) $C(a,p) \rightarrow B(a,p).$

The validity of this principle derives from the fact that each proposition *p* with maximal probability 1 a fortiori has a probability greater than .5! Thus, *semantically* speaking, *a*’s believing that *p* is entirely compatible with *a*’s being absolutely certain that *p*, although from a *pragmatic* point of view when person *a* says ‘I believe that *p*’, she thereby expresses that she is *not convinced* that *p*.

The epistemological thesis of the privileged access to (or privileged knowledge of) our own mental states mentioned earlier in connection with principles **E1** and **E2** evidently applies not only to the particular doxastic attitude $C(a,p)$, but to the more general notion $B(a,p)$ as well. Thus, whenever person *a* believes that *p*, *a* knows that she believes that *p*; and, conversely, if she does not believe that *p*, she knows that she does not believe that *p*:

(E10) $B(a,p) \rightarrow K(a,B(a,p))$

(E11) $\neg B(a,p) \rightarrow K(a,\neg B(a,p)).$

In view of **E3** and **E8** one immediately obtains the following pure iteration-laws:

(B2) $B(a,p) \rightarrow B(a,B(a,p))$

(B3) $\neg B(a,p) \rightarrow B(a,\neg B(a,p)).$

Furthermore the rules of deductive closure of belief:

(B4) $p \leftrightarrow q \vdash B(a,p) \leftrightarrow B(a,q)$

(B5) $p \rightarrow q \vdash B(a,p) \rightarrow B(a,q)$

(B6) $p \vdash B(a,p)$

can be justified in strictly the same way as the corresponding principles for conviction or for knowledge.

In order to obtain a complete axiomatization of the logic of („weak") belief, one first has to introduce the relation of "strict implication" between *sets of propositions* $\{p_1,...,p_n\}$ and $\{q_1,...,q_n\}$ ($n\geq 2$). Let this generalization of the ordinary relation of logical implication be symbolized by $\{p_1,...,p_n\} \Rightarrow \{q_1,...,q_n\}$. This relation has been defined by Segerberg [1971] to hold iff, for logical reasons, at least as many propositions from the set $\{p_1,...,p_n\}$ must be true as there are true propositions in the set $\{p_1,...,p_n\}$. Now, just like the logical implication between p and q guarantees that the probability of q is at least as great as the probability of p, so also the strict implication between $\{p_1,...,p_n\}$ and $\{q_1,...,q_n\}$ entails that the *sum* of the probabilities of the q_1 is at least as great as the corresponding sum $\sum_{i\leq n}$ Prob(a, p_i). Therefore, if at least one proposition from $\{p_1,...,p_n\}$ is believed by a to be true (and hence has a probability > .5) and if all the other p_i are not believed by a to be false (and hence have a probability ≥ .5), so that in sum $\sum_{i\leq n}$ Prob(a, p_i) > n•1/2, it follows that also $\sum_{i\leq n}$ Prob(a,q_i) > n/2, and thus at least one of the q_i must be believed by a to be true:

(B7) $$\{p_1,...,p_n\} \Rightarrow \{q_1,...,q_n\} \vdash B(a,p_1) \wedge \neg B(a,\neg p_2) \wedge ... \wedge \neg B(a,\neg p_n) \rightarrow$$
$$\rightarrow B(a,q_1) \vee ... \vee B(a,q_n).$$

4 The Pragmatics of Epistemic Sentences

The foregoing sketch of the logics of knowledge, belief, and conviction may be considered as an investigation into the *truth-conditions* of epistemic attitudes. Let us now turn to the *conditions of utterance* (or utterability) of corresponding sentences. This investigation is meant to explain, among others, why sentences of the type '*a* mistakenly believes that *p*' or '*a* is convinced that *p*, but *a* does not know that *p*' can only be reasonably maintained by a speaker $b \neq a$. Also we want to explain why statements of the form 'I think it possible that *p*' or 'I believe that *p*' are normally understood to mean (in addition to what is explicitly asserted) that, in the first case, the speaker does not believe that *p*, or, in the second case, that the speaker is not convinced that *p*. These particular aspects of the pragmatics of *epistemic* sentences shall not be given ad hoc, however, but rather be derived from a general theory of utterances of arbitrary sentences. Such a general pragmatics will rely on certain maxims of (rational) communication or conversation, which in turn has to be rendered precise by means of epistemic logical principles.

As a first, very elementary principle of rational communication consider the so-called "maxim of quality" which was formulated in the classical study Grice [1975: 46] as follows: "Do not say what you believe to be false!". This maxim is in need of some modifications. First, instead of requiring 'Do not *say* what you believe to be false' it would be more adequate to demand 'Do not *assert* what you believe to be false', because when people are using language, e.g., in an ironic way they often *say p* in order to *assert* ¬*p*. Second, it would be better to transform Grice's doubly negated maxim („not ... false") into the positive maxim 'Only assert what you believe to be true!'. Otherwise, if *p* is a proposition such that person *a* neither believes that *p* nor believes that ¬*p*, (e.g., *p* = 'Throwing this coin will show up heads') it would count as rational for *a* to assert both *p* and ¬*p*. Third, the question arises in which sense or to which degree *a* has to believe that *p* in order to be entitled to rationally assert *p*. Is it sufficient that *a* just believes *p* in the ordinary, weak sense of $B(a,p)$, or should it be required that *a* must strongly believe *p* in the sense of $C(a,p)$?

Our communicative practice in everyday life shows that the hearer in general takes it for granted that the speaker is certain or convinced of what he asserts. Gazdar [1979: 46] suggested an even stronger variant of the maxim of quality: "Say only that which you *know*". But this requirement is too demanding because, for epistemic-logical reasons, the speaker could never know whether he really satisfies it. Therefore it appears reasonable to demand only that the speaker must always *think* or *believe* to know that what he asserts is true. Using

‘$A(a,p)$’ to abbreviate ‘a asserts that p’, this version of the maxim of quality can be formalized as follows:

(**QUAL**) $A(a,p) \rightarrow_p C(a,p)$.

In contrast to the symbol ‘$\rightarrow$’ for ordinary logical implication, the new sign ‘$\rightarrow_p$’ is meant to symbolize a relation of *pragmatic* implication which should be understood in some such way that **QUAL** reads as follows: ‘If a asserts p, a thereby shows (or indicates) that he is convinced that p’; or ‘The fact that a asserts p entitles the hearer to infer that a is certain that p’; or also ‘By the maxims of (rational) conversation, a is entitled to assert p only if a is convinced that p’, etc.

Grice [1975: 45] has put forward another important maxim (called the “maxim of quantity”) which says: “Make your contribution as informative as is required (for the current purposes of the exchange)”. This principle also is in need of some clarifications and modifications. First it should be observed that the maxim of quantity may be incompatible with the maxim of quality ‘Say only that which you believe to know’. The recommendation to make your contribution as informative as is required has to be restricted by the clause ‘as far as you can’. We often face situations where the others expect us to say more than we know (or, more precisely, more than we believe to know). Second, even if we have the desired knowledge at hand, it is not always clear *how informative* our contributions should be and to which extent the others want to be informed about details of the matter. Unfortunately, there does not appear to exist a meaningful *quantitative* notion of (the degree of) information. Indeed, unlike in the case of probabilistic concepts, we do not even have a clear *classificatory* concept of informational content at hand. At best we have some intuitions about a *comparative* concept of information. Thus it is plausible to assume that p is more informative than q only if p has greater logical content than q, i.e. only if p logically entails q. Conversely, however, p’s logically entailing q does not always mean that p is (in any *relevant* aspect) more informative than q, for the extra content by which p surpasses q may be entirely irrelevant for the purposes of the conversation. Since there is no clear and immediate logical definition of informational content in sight, let us presuppose the relation ‘p is (relevantly) more informative than q’ as primitive here, and let it be symbolized by ‘$p >_i q$’.

The basic idea of Grice’s maxim of quantity can now be rendered more precise in the following way. If the speaker a has the choice between two assertions p and q, where the former is relevantly more informative than the latter, then a is conversationally obliged to make the more informative assertion p provided this is not in conflict with the maxim of quality. In other words, a is conversationally allowed to make the less informative assertion q only if a is *not certain* that p. To put it formally:

(**QUAN**) If $(p >_i q)$, then $A(a,q) \rightarrow_p \neg C(a,p)$.

This pragmatic implication may be combined with the former **QUAL** so as to yield the following *main principle of pragmatics*: By making the weaker or less informative assertion q, the speaker pragmatically implies or indicates that while she is convinced that q, she is not certain that (and hence does not know whether) the more informative p also is the case:

(**PRAG 1**) If $(p >_i q)$, then $A(a,q) \rightarrow_p C(a,q) \wedge \neg C(a,p)$.

Let us now apply this general pragmatic principle to utterances of certain *epistemic* propositions p and q in particular and assume that the logical relations existing between the epistemic attitudes of some person b correlate with corresponding degrees of (relevant) informational content. Thus, e.g., sentence $K(b,p)$ may be taken to be more informative than $C(b,p)$ which in turn is $>_i B(b,p)$, and again $B(b,p) >_i P(b,p)$. One thus obtains the following corollaries:

(**PRAG 2**) $A(a,P(b,p)) \rightarrow_p C(a,P(b,p)) \wedge \neg C(a,B(b,p))$

(**PRAG 3**) $A(a,B(b,p)) \rightarrow_p C(a,B(b,p)) \wedge \neg C(a,C(b,p))$

(**PRAG 4**) $A(a,C(b,p)) \rightarrow_p C(a,C(b,p)) \wedge \neg C(a,K(b,p))$.

Now consider the special instances where speaker a makes assertions about his own epistemic attitudes. Setting $b = a$ in **PRAG 2** we obtain that if a asserts that he considers it possible that p, he pragmatically implies $C(a,P(a,p)) \wedge \neg C(a,B(a,p))$.In view of the earlier laws for iterated doxastic attitudes, this may be simplified as follows:

(**PRAG 5**) $A(a,P(a,p)) \rightarrow_p P(a,p) \wedge \neg B(a,p)$.

By asserting that he considers it possible that p, a indicates that he does not believe that p (and a fortiori that he does not know that p). In this sense a's *utterance* of $P(a,p)$ is *pragmatically* incompatible with his (weakly or strongly) believing that p or with his knowing that p, even though - as we have seen above - the state of affairs $P(a,p)$ is *semantically* (or also objectively) compatible with $B(a,p)$, $C(a,p)$, and $K(a,p)$.

Similarly it follows from **PRAG 3** that if a asserts that he believes that p, he thereby pragmatically implies that he is not convinced that p:

(**PRAG 6**) $A(a,B(b,p)) \rightarrow_p B(a,p) \wedge \neg C(a,p)$.

Many controversies in the literature concerning the validity of epistemic-logical laws - and especially a great part of the discussion of the so-called “Entailment-thesis” $K(a,p) \rightarrow B(a,p)$) - suffer from not sufficiently distinguishing between semantic and pragmatic implications. Once this distinction is made, one not only obtains a perfectly consistent theory of propositional attitudes but actually one in which the semantic laws serve to *explain* the pragmatic ones.

Turning now to **PRAG 4** we face a problem, however. If one would just substitute b for a in **PRAG 4,** one would obtain the principle $A(a,C(a,p)) \rightarrow_p C(a,C(a,p)) \wedge \neg C(a,K(a,p))$ which is intuitively unacceptable. It says that if person a asserts that she is certain that p, a thereby pragmatically implies not only (as desired) $C(a,C(a,p))$, i.e. $C(a,p)$, but also $\neg C(a,K(a,p))$, i.e., in view of **E5**, $\neg C(a,p)$. In other words, a person who asserts ‘I am certain that p’ would imply that she does not know that p and hence that she is *not* certain that p, after all.

To solve this problem, observe that **PRAG 4** is based on the premise that asserting ‘b knows that p’ is more informative than asserting ‘b is convinced that p’. This assumption certainly is justified in the general case where $b \neq a$. Since $K(b,p)$ unlike $C(b,p)$ semantically entails p, a's assertion of ‘b *knows* that p’ is more informative than a's assertion of ‘b is *convinced* that p’ because the former pragmatically entails not only $C(a,C(b,p))$ but in addition also $C(a,p)$. Thus by asserting ‘b *knows* that p’ person a indicates that she herself is convinced that p, too. However, in the special case $b = a$, the corresponding assertion of ‘a knows that p’ (or, more idiomatically, ‘I know that p’) is no longer more informative than ‘I am certain that p’ (and, incidentally, no more informative than the simple assertion of p, either). In view of **QUAN** both $A(a,p) \rightarrow_p C(a,p)$, and $A(a,C(a,p)) \rightarrow_p C(a,C(a,p))$, and also $A(a,K(a,p)) \rightarrow_p C(a,K(a,p))$; but for epistemic logical reasons the respective pragmatic consequents $C(a,p)$, $C(a,C(a,p))$, and $C(a,K(a,p))$ are all *equivalent* to each other. Thus no matter whether one simply asserts ‘p’, or ‘I am convinced that p’ or ‘I know that p’, the pragmatic effects on the hearer h will always be the same: h may take it for granted that speaker a is convinced that p, but this cannot logically warrant that p actually is the case. Therefore the premise $K(a,p) >_i C(a,p)$ does not hold when a's own utterances are at stake, and so in the case $a = b$ the maxim **QUAL** must not be applied.

The pragmatic principles developed above may further be used to explain several phenomena that have been observed in connection with epistemic utterances. So-called Moore's paradox consists in the fact that one cannot rationally assert, e.g., 'It is raining, but I do not believe that'. The reason of the *pragmatic inconsistency* of this assertion (which, when analyzed from a semantic points of view, describes a fully self-consistent state of affairs) lies in principle **QUAN**. By asserting '$p\wedge\neg B(a,p)$', person a pragmatically implies $C(a,p\wedge\neg B(a,p))$ which is epistemic-logically equivalent to $C(a,p) \wedge C(a,\neg B(a,p))$, i.e. to $C(a,p) \wedge \neg B(a,p)$, in flat contradiction to principle **E9**. The solution of the so-called "Surprise-Examination" or "Hangman-Paradox" also crucially depends on the pragmatic implications of epistemic sentences, but it requires a much subtler analysis that cannot be given in the framework of this paper.[6]

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[6] Cf., e.g., Lenzen [1976] and ch. 5 of Lenzen [1980].

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