Logical Criteria for Individual(concept)s

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1 Introduction

According to an oft-quoted passage from the *Discourse on Metaphysics*, the complete concept of an individual "suffices to include and to render deducible from it all the predicates of the subject to which this notion is attributed"[1]. This formulation gives rise to a number of questions:

(A) Does this condition define a real distinction between individual concepts and other concepts?

(B) How can Leibniz's condition be rendered logically precise?

(C) Are these logical reconstructions compatible with Leibniz's remaining views on individual(concept)s?

(D) Did Leibniz himself develop precise logical criteria (or at least come close to such criteria) for the completeness of individual concepts?

Issue (B) will be dealt with in section 2 below where some contemporary reconstructions of the completeness condition shall be considered. In order to answer questions (C) and (D), it will be necessary first to outline Leibniz's logic in section 3 before his own criteria for individual(concept)s can be investigated in section 4. Right now, however, let me address question (A) and clarify in which sense individual concepts differ from ordinary concepts.

Rutherford [1988: 131] has argued that not only individual concepts, but also ordinary concepts such as the concept *man* satisfy the crucial completeness condition. For one "may reason that the concept *man* must include the concepts of all and only those predicates which are truly attributed to man, and infer that *man* is in this sense complete." This argument appears to be based on the following premises.

(i) Concept A contains concept B if and only if ('iff', for short) – in terms of the traditional theory of the syllogism – Every A is B.

(ii) A predicate P is "truly attributed" to a subject S iff – in terms of modern (predicate) logic – $\forall x\ (S(x) \rightarrow P(x))$. Therefore, clearly, whenever B is "truly attributed" to A, i.e. whenever $\forall x\ (A(x) \rightarrow B(x))$, then every A is B and hence concept A contains B.

Now, for Leibniz the formula 'A is B' [A est B] or 'A contains B' [A continet B] always has to be understood as the *universal* affirmative proposition *Every A is B*. Hence Rutherford's premise (i) is warranted. However, as regards premise (ii), there are two different ways in which a predicate term (or concept) B may be "truly attributed" to the subject term (or concept) A. B is *universally* attributed to A just in case that every A is B, $\forall x\ (A(x) \rightarrow B(x))$, while B is *particularly* attributed to A if *some* A is B: $\exists x(A(x) \wedge B(x))$. Thus Rutherford's conclusion is correct only insofar as for each ordinary concept A, A contains B iff the predicate concept B can be *universally* attributed to the subject A. But, clearly, A will normally not contain all predicates B which can be *particularly* attributed to it because, if some A is B, there will often exist also some A which is not B, so that A would otherwise have to contain both B and its negation, Non-B, and hence be inconsistent.

In § 71 GI Leibniz stressed this point as follows: "If you say that some man is a denier, [concept] *man* does not contain *denial* since it is an incomplete term".[2] What does it, however, mean that, unlike ordinary concepts, an individual concept as, e.g., the concept

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1 Cf. A VI, 4, 1540, translation from Rutherford [1988: 132]; many related passages may be found especially in the Arnauld correspondence, cf. Sleigh [1990].

2 Cf. A VI, 4, 762: "et hoc discrimen est inter terminum individuum seu completum, et alium; nam si dicam aliquis homo est abnegans, homo non continet abnegationem, est enim terminus incompletus".

Apostle Peter does contain all predicates B which can be particularly (or individually) attributed to it? At the beginning of the fragment on "Some logical difficulties", Leibniz gave the following hints:

"How is it that opposition is valid in the case of singular propositions – e.g., 'The Apostle Peter is a soldier' and 'The Apostle Peter is not a soldier' – since elsewhere a universal affirmative and a particular negative are opposed. Should we say that a singular proposition is equivalent to a particular and a universal proposition? Yes, we should. So also when it is objected that a singular proposition is equivalent to a particular proposition, since the conclusion in the third figure must be particular, and can nevertheless be singular; e.g., 'Every writer is a man; some writer is the Apostle Peter; therefore the Apostle Peter is a man'. I reply that here also the conclusion is really particular, and it is as if we had drawn the conclusion 'Some Apostle Peter is a man'. For 'some Apostle Peter' and 'every Apostle Peter' coincide, since the term is singular." (Cf. GP VII, 211; translation from Parkinson [1966: 115]).

In the concluding sentence Leibniz puts forward the "Wild quantity thesis"[3] which certainly must appear very strange to modern logicians. It maintains that a singular proposition of the form 'C is B' is equivalent both to the universal proposition 'Every C is B' and to the particular proposition 'Some C is B'. As will be shown below, despite their prima facie implausibility, these principles are *provable* within the system of Leibniz's logic. As a matter of fact, the formal counterpart of the first "half" of the "Wild quantity thesis" turns out to be a general theorem even for "ordinary" concepts, C, while the formal counterpart of the second "half" holds only for individual concepts. Before we turn to these issues, however, let us consider several other conditions for complete individual concepts as they have been discussed by modern Leibniz scholars.

2 Contemporary reconstructions of the "completeness" condition

According to Mates [1986: 62-63], the "metaphysical" characterization "In the perfect concept of each individual substance is contained all its predicates, both necessary and contingent, past, present, and future" (cf. A VI, 4, 1618) may be interpreted quite straightforwardly as saying that "a complete individual concept contains every attribute of every individual that can fall under it, […] that is, it determines exactly one possible individual". Hence, on the one hand, one obtains a one-to-one correspondence between a (possible) *individual*, c, and its individual-*concept*, C: For each possible individual c there exists a complete concept C such that C fully characterizes c (and c only). On the other hand, the crucial "characterization" of c might be paraphrased in terms of modern first order predicate logic as follows:

(IND 1) C is the complete concept of the individual c iff for every concept (or attribute) B: C *contains* B iff B(c).

If one would furthermore follow Sleigh [1990: 49] and interpret the condition "C contains B" as the set-theoretical $\in$-relation, one might define, for any individual c, c's complete individual concept, C, as the *set* "C whose members are all and only the […] properties of [c]":

(IND 2) C is the complete concept of the (possible) individual c iff C = {B: B(c)}.

At first sight, this definition appears to accord quite well with Leibniz's own views as expressed, e.g., in LH IV, VII C 103-4:

3 As it has been dubbed by Sommers [1976]; cf. also Englebretsen [1988].

"If term A involves all terms B, C, D, etc. that can be said of the same thing, term A expresses a singular substance; i.e., the concept of a singular substance is a complete term, containing everything that can be said of that substance".

However, there are at least two reasons why IND 1, 2 do not adequately reflect Leibniz's logical conception of an individual's complete concept. First, the relation of "involvement" or *containment* between concepts must not be modelled by the set-theoretical $\in$-relation, but rather by set-theoretical containment, $\subseteq$[4]. Second, the logical relation of *predication* (as it is somewhat hidden behind the brackets of the formula B(c)), is not, as such, available for Leibniz. Leibniz never makes any syntactic distinctions between a singular term and a general term, nor does he see any structural differences between a universal predication like '(Every) Man is mortal' and a singular predication like 'Caesar is mortal'. Both sentences, for Leibniz, have one and the same logical form: 'C is B', or 'C contains B'. In what follows this fundamental relation of conceptual containment shall be abbreviated by CεB. Hence in Leibniz's logic the predication B(c) must equally be represented by CεB, so that the decisive criterion of IND 1, "C contains B iff B(c)", becomes the undiscriminating tautology: CεB iff CεB! Let us therefore consider three other interpretations of the completeness condition which better reflect Leibniz's views of individuals.

In her pioneering study *Über die Leibnizsche Logik*, Raili Kauppi briefly discussed a particular feature of Leibniz's logic – so-called "indefinite concepts" – and their relation to complete individual concepts:

"In that calculus [i.e., in the GI] letters from the beginning of the alphabet are symbols for definite concepts, letters from the end of the alphabet symbols for indefinite concepts. The expression 'AY' therefore means an indefinite kind of the species A and it corresponds to the expression 'a certain A' [*quoddam A*]. The same variables are also used for individuals which are introduced into the calculus in the sense of complete individual concepts. If for arbitrary Y the coincidence BY = B holds (provided BY is no contradiction) then B is an individual. This means that concept B cannot be expanded by any new property without generating a contradiction." (Cf. Kauppi [1960: 168]; my translation).

In the last sentence Kauppi suggests to interpret individual concepts as *maximally consistent* concepts which "cannot be expanded by any new property without generating a contradiction", and she evidently takes it for granted that the corresponding formal requirement:

(IND 3) B is an individual concept iff, for arbitrary Y the coincidence BY = B holds (provided BY is no contradiction)[5]

adequately captures Leibniz's own ideas as expressed in § 72 GI:

(72) So if we have BY, and the indefinite term Y is superfluous (i.e., in the way that 'a certain Alexander the Great' and 'Alexander the Great' are the same), then B is an individual. If there is a term BA and B is an individual, A will be superfluous; or if BA=C, then B=C." (Parkinson [1966: 65])

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4 Or, more exactly, by the reverse relation $\supseteq$; see section 3 below.

5 Basically the same interpretation has been adopted by H. Burkhardt and by F. Schupp. Burkhardt [1980: 347] remarks: „Heute würden wir von maximal konsistenten Begriffen sprechen und dies folgendermaßen symbolisieren: B ist ein Individuum = $\Lambda Y(cons(BY) \rightarrow Y \subseteq B)$". Similarly Schupp [1982: 182] points out: "Ein erschöpfender Begriff ist dadurch gekennzeichnet, daß er durch keine weiteren Bestandteile vermehrt werden kann. Daher ist z.B. ‚Alexander der Große' – B und ‚ein bestimmter Alexander der Große' – BY dasselbe. Kann daher zu einem Begriff ein beliebiger unbestimmter hinzugefügt werden, und ist jeder solcher hinzugefügter Begriff überflüssig, so bezeichnet dieser Begriff ein Individuum."

Quite a different interpretation of the completeness condition has been suggested by Mates [1986: 63]. Referring to a series of definitions where Leibniz explains that there exist as many singular substances as there are different combinations of all the compatible attributes ["tot esse substantias singulares quot sunt diversae combinations omnium attributorum compatibilium",A VI, 4, 306-308], Mates believes to see the textual basis for the following criterion: "a complete individual concept is a concept that contains, for every simple attribute, either that attribute or its negation, but not both". If we symbolize the negation of a concept A by $\overline{A}$, this interpretation can be formalized as follows:

(IND 4) B is an individual concept iff, for every concept Y: Either B contains Y or B contains $\overline{Y}$.

Furthermore, Mates [1986: 63; fn. 57] hints to yet another characteristic of complete individual concepts. In the fragment "Notationes Generales" (A VI, 4, 550-557) Leibniz explains that a complete concept "cannot be the predicate term in a true proposition unless that proposition is reciprocal, as in, e.g., 'Peter is the apostle who denied Christ' and 'The apostle who denied Christ is Peter'." Generalizing from this example one would have to postulate:

(IND 5) B is an individual concept iff, whenever B is contained in some concept Y, B conversely contains Y (and hence is identical with Y).

Now it remains far from clear whether these criteria really "work" and whether they are warranted by Leibniz's own considerations. First, it is not at all trivial to see whether IND 3 - IND 5 are provably *equivalent* to each other. Second, Leibniz nowhere appears to have considered the important condition of the *consistency* of BY as introduced by Kauppi in principle IND 3. Third, one may doubt whether the *universal* quantifier in IND 3 adequately captures Leibniz's requirement of the "redundancy" of Y as put forward in § 72 GI. After all, the example of 'a *certain* Alexander the Great' and 'Alexander the Great' contains an *existential* rather than a universal quantifier. Anyway, it remains to be checked quite carefully how all this relates to the "Wild quantity thesis" mentioned in section 1 above.

To conclude this short overview of modern interpretations of the completeness condition let me just quote a passage from Liske [2000: 70] which shows that the interpretation of complete individual concepts as maximal consistent concepts has eventually become a commonplace among Leibniz scholars:

„Ein Individualbegriff ist demgegenüber ein maximal bestimmter Begriff; zu ihm kann kein weiterer Begriffsinhalt mehr hinzugefügt werden, jede Hinzufügung wäre entweder inkonsistent oder aber überflüssig. [...] Damit kann er nicht Teil eines anderen, spezielleren Begriffs darstellen, er ist der speziellste Begriff, der genau ein mögliches Individuum festlegt."

In the remainder of this paper I want to show that this interpretation is not only compatible with Leibniz's "metaphysical" views of individuals but even derivable from the basic laws of his own logic.

3 An Outline of Leibniz's Logic

Although during his lifetime Leibniz never developed a final, complete system of logic, there are two main calculi which can be clearly reconstructed from his logical fragments: the algebra of concepts which I shall call *L1*, and the extension of this system by so-called

indefinite concepts, *L2*.[6] The starting point of Leibniz's logic is the traditional "Aristotelian" theory of the syllogism with the four categorical forms of universal or particular, affirmative or negative propositions:

U.A.	Every A is B	U.N.	No A is B
P.A.	Some A is B	P.N.	Some A is not B

Within the framework of so-called "Scholastic" syllogistics, negative concepts Not-A are also taken into account, which are here symbolized by $\overline{A}$.

The algebra *L1* as developed by Leibniz in some early fragments of around 1679 and above all in the GI of 1686 grows out of this syllogistic framework by three achievements. First, Leibniz drops the expression 'every' ["omne"] and formulates the U.A. simply as 'A is B' ["A est B"] or also as 'A contains B' ["A continet B"]. As was already explained in the introduction, this fundamental proposition shall here be symbolized as ‚AεB'. Second, Leibniz introduces the new operator of conceptual *conjunction* which combines two concepts A and B by juxtaposition to AB. Third, Leibniz disregards all traditional restrictions concerning the number of premises and concerning the number of concepts in the premises of a syllogism. Thus arbitrary inferences between sentences of the form AεB will be taken into account, where A and B may contain negations and conjunctions of other concepts.

In order to axiomatize *L1* one may chose (besides the tacitly presupposed propositional functions $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$) only negation, conjunction and the ε-relation as primitive conceptual operators. As regards the relation of conceptual containment, AεB, it is important to observe that Leibniz's formulation 'A contains B' pertains to the so-called *intensional* interpretation of concepts as *ideas*, while we here want to develop an *extensional* interpretation in terms of *sets of individuals*, *viz*. the sets of all individuals that fall under the concepts A and B, respectively. Leibniz explained the mutual relationship between the "intensional" and the extensional point of view in the following passage of the *New Essays on Human understanding*:

"The common manner of statement concerns individuals, whereas Aristotle's refers rather to ideas or universals. For when I say *Every man is an animal* I mean that all the men are included amongst all the animals; but at the same time I mean that the idea of animal is included in the idea of man. 'Animal' comprises more individuals than 'man' does, but 'man' comprises more ideas or more attributes: one has more instances, the other more degrees of reality; one has the greater extension, the other the greater intension" (cf. A VI, 1: 486; my translation).

In Lenzen [1983] precise definitions of the intension and the extension of concepts have been developed which show that Leibniz's intensional point of view becomes provably equivalent to the more common set-theoretical point of view, provided that the extensions of concepts are taken from a universe of discourse, U, to be thought of as a set of *possible individuals*. In particular, the intensional proposition AεB, according to which concept A *contains* concept B, has to be interpreted extensionally as saying that the set of all A's *is included* in the set of all B's.

Next consider the identity or *coincidence* of two concepts which Leibniz usually symbolizes by the modern sign '=' or by the symbol '∞', but which he sometimes also refers to informally by speaking of two concepts being the same ["idem", "eadem"]. As stated, e.g., in § 30 GI, identity or coincidence can be defined as mutual containment: "That A is B et B is A is the same as that A and B coincide".

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6 Furthermore, Leibniz made many important discoveries in the field of propositional logic, alethic modal logic, and deontic logic, but these will be ignored here. Cf. Lenzen [1987] and [2001].

In most drafts of the "universal calculus", Leibniz symbolizes the operator of conceptual *conjunction* by mere juxtaposition in the form AB. The intended interpretation is straightforward. The extension of AB is the set of all (possible) individuals that fall under both concepts, i.e. which belong to the *intersection* of the extensions of A and of B.

Let it be noted in passing that the above condition for the interpretation of ε (which reflects the reciprocity of extension and intension) would be derivable from the interpretation of identity and conjunction, if the ε-relation were defined according to § 83 GI in terms of the latter: "Generally, 'A is B' is the same as 'A=AB'".

The next element of the algebra of concepts – and, by the way, one with which Leibniz had notorious difficulties – is *negation*. Leibniz usually expressed the negation of a concept by means of the same word he also used to express propositional negation, viz. "non". Especially throughout the GI, the statement that one concept, A, contains the negation of another concept, B, is expressed as 'A is not-B' ["A est non B"], while the related phrase 'A isn't B' ["A non est B"] has to be understood as the mere negation of 'A contains B'. As was shown in Lenzen [1986], during the whole period of the development of the "universal calculus" Leibniz had to struggle hard to grasp the important difference between 'A is not-B' and 'A isn't B'. Again and again he mistakenly identified both statements, although he had noted their non-equivalence repeatedly in other places. Since the negation of concept A is here expressed as $\overline{A}$, while propositional negation is symbolized by means of the usual ¬, 'A is not-B' takes the form $A\varepsilon\overline{B}$, while 'A isn't B' has to be rendered as ¬AεB. The intended extensional interpretation of $\overline{A}$ is just the set-theoretical complement of the extension of A, because each individual which fails to fall under concept A eo ipso falls under the negative concept $\overline{A}$.

Closely related with the operator of negation is that of *possibility* or self-consistency of concepts. Leibniz expresses it in various ways. He often says 'A is possible' ["A est possible"] or 'A is [a] being' ["A est Ens"] or also 'A is a thing' ["A est Res"]. Sometimes the self-consistency of A is also expressed elliptically by 'A est', i.e. 'A is'. Here a capital 'P' will be used to abbreviate the possibility of a concept. According to GI, lines 330-331, the operator P can be defined as follows: "*A* not-*A* is a contradiction. *Possible* is what does not contain a contradiction or *A* not-*A*": $P(B) \leftrightarrow \neg(B\varepsilon A\overline{A})$.[7]

It then follows from our earlier conditions that P(A) is true (under an extensional interpretation ϕ) if and only if the extension of concept A, ϕ(A), is *not empty*. At first sight, this condition might appear inadequate, since there are certain concepts – such as that of a unicorn – which happen to be empty but which may nevertheless be regarded as possible, i.e. not involving a contradiction. Remember, however, that the universe of discourse underlying the extensional interpretation does not consist of *actually existing* objects only, but instead comprises all *possible* individuals. Therefore the non-emptiness of the extension of A is both *necessary and sufficient* for guaranteeing the self-consistency of A. Clearly, if A is possible then there must exist at least one *possible* individual that falls under concept A.

The main elements of Leibniz's algebra of concepts may thus be summarized in the following diagram.

Element of the Algebra of Concepts L1	*Symbolization*	*Leibniz's Notation*	*Set-theoretical Interpretation*
Identity	A=B	A∞B; A=B; coincidunt A et B; ...	$\phi(A) = \phi(B)$
Containment	AεB	A est B; A continet B	$\phi(A) \subseteq \phi(B)$
Conjunction	AB	AB; A+B	$\phi(A) \cap \phi(B)$

7 This definition might be simplified as follows: $P(B) \leftrightarrow_{df} \neg(B\varepsilon\overline{B})$.

Negation	$\overline{A}$	Non-A	$\overline{\phi(A)}$
Possibility	P(A)	A est Ens; A est res; A est possibile	$\phi(A) \neq \emptyset$

Let's now have a brief look at some *fundamental theorems* of *L1* all of which (with the possible exception of the last one) were stated by Leibniz himself:

Theorems of L1	*Formal version*	*Leibniz's version*
CONT 1	$A\varepsilon A$	"B is B" (GI, § 37)
CONT 2	$A\varepsilon B \wedge B\varepsilon C \rightarrow A\varepsilon C$	"[...] if A is B and B is C, A will be C" (GI, § 19)
CONT 3	$A\varepsilon B \leftrightarrow A{=}AB$	"Generally '*A* is *B*' is the same as '*A* = *AB*'" (GI, § 83)
CONJ 1	$A\varepsilon BC \leftrightarrow A\varepsilon B \wedge A\varepsilon C$	"That A contains B and A contains C is the same as that A contains BC" (GI, § 35)
CONJ 2	$AB\varepsilon A$	"AB is A" (A VI, 4, 813)
CONJ 3	$AB\varepsilon B$	"AB is B" (GI, § 38)
CONJ 4	$AA = A$	"AA = A" (GI, § 171, Third)
CONJ 5	$AB = BA$	"AB ∞ BA" (C. 235, # (7))
NEG 1	$\overline{\overline{A}} = A$	"Not-not-A = A" (GI, § 96)
NEG 2	$A \neq \overline{A}$	"A proposition false in itself is 'A coincides with not-A'" (GI, § 11)
NEG 3	$A\varepsilon B \leftrightarrow \overline{B}\,\varepsilon\,\overline{A}$	"In general, 'A is B' is the same as 'Not-B is not-A'" (GI, § 77)
NEG 4	$\overline{A}\,\varepsilon\,\overline{AB}$	"Not-A is not-AB" (GI, § 76a)
NEG 5	$[P(A) \wedge]\ A\varepsilon B \rightarrow \neg(A\varepsilon\overline{B})$	"If A is B, therefore A is not not-B" (GI, § 91)
POSS 1	$\neg P(A\overline{B}) \leftrightarrow A\varepsilon B$	"if I say 'A not-B is not', this is the same as if I were to say [...] 'A contains B'" (GI, § 200).[8]
POSS 2	$A\varepsilon B \wedge P(A) \rightarrow P(B)$	"If A contains B and A is true, B is also true" (GI, § 55)[9]
POSS 3	$\neg P(A\overline{A})$	"A not-A is not a thing" (GI, § 171, Eighth)
POSS 4	$A\overline{A}\,\varepsilon B$[10]	

8 Parkinson translates Leibniz's "Si dicam AB non est ..." somewhat infelicitous as "If I say ‚AB does not exist'..." thus blurring the distinction between (actual) *existence* and mere *possibility*. For an alternative formulation of POSS 1 cf. A VI, 4 863: „[…] si *A est B* vera propositio est, *A non-B* implicare contradictionem", i.e. 'A is B' is a true proposition if A non-B includes a contradiction.

9 At first sight this quotation might seem to express some law of propositional logic such as *modus ponens*: If A→B and A, then B. However, as Leibniz goes on to explain, when applied to concepts, a "true" term is to be understood as one that is self-consistent: "[...] By a false letter I understand either a false term (i.e. one which is impossible, or, is a non-entity) or a false proposition. In the same way 'true' can be understood as either a possible term or a true proposition" (ibid.).

10 POSS 4 is the counterpart of what one calls "ex contradictorio quodlibet" in propositional logic: An inconsistent concept contains every other concept! Although this law has never been stated explicitly by Leibniz, it may nevertheless be regarded as a Leibnizian theorem because it follows from POSS 1, 2 and 3. Furthermore in GP VII, 224/5 Leibniz explained: "[...] the round square is a quadrangle with null-angles. For this proposition is true in virtue of an impossible hypothesis". The text-critical apparatus in A VI, 4, 293 reveals that he had originally added: "Nimirum de impossibile concluditur impossibile". So in a certain way he was aware of the principle "ex contradictorio quodlibet".

As was shown in Lenzen [1984b: 200], the set of principles {CONT 1, CONT 2, CONJ 1, NEG 1, POSS 1, POSS 2} already provides a complete axiomatization of the algebra of concepts which is isomorphic to the Boolean algebra of sets.

Let us now briefly consider an important extension of *L1* by way of introducing so-called "indefinite concepts". In many logical fragments Leibniz uses letters from the end of the alphabet (x, y, ..., X, Y, Z, ...) and occasionally also from the mid of the alphabet (Q, L, ...) for the representation of „indefinite concepts", while the „normal" concepts are symbolized by letters from the beginning of the alphabet.[11] As has been shown in Lenzen [1984a]

a) indefinite concepts primarily function as (existential and universal) *quantifiers ranging over concepts*;
b) Leibniz somehow "felt" the difference between indefinite concepts functioning as existential quantifiers and those functioning as universal quantifiers, but his elliptic formalization fails to bring out this difference with sufficient clarity and precision;
c) Leibniz nevertheless anticipated some fundamental laws of quantifier logic and may thus be considered at least as a forerunner of modern quantification theory.

For reasons of space only the bare essentials of Leibniz's theory of indefinite concepts can be outlined here. In § 16 GI the "*Affirmative Proposition* A is B" is analyzed as follows:

"[...] That is, if we substitute a value for A, ‚A coincides with BY' will appear [...] For by the sign Y I mean something undetermined, so that BY is the same as some B [...] So 'A is B' is the same as 'A is coincident with some B', or A=BY."[12]

This principle, according to which AεB is equivalent to A = BY, has to be interpreted more exactly as the existentially quantified proposition that A contains B if and only if *there exists some Y such that* A = BY:

(CONT 4) $A\varepsilon B \leftrightarrow \exists Y(A=BY)$.

This explicit introduction of the existential quantifier not only accords with Leibniz's own intentions but it was also anticipated by him in some other fragments. Thus in § 10 of "The Primary Bases of a Logical Calculus" (C. 235-7) he used the expression "there can be assumed a Y such that A = YB" (Parkinson [1966: 90]). And in the "Specimina Calculi Rationalis" Leibniz starts by putting forward the law

(NEG 6*)[13] $\neg(A\varepsilon B) \leftrightarrow \exists Y(YA\varepsilon \overline{B})$

elliptically as "A is not B is the same as QA is non B" ["*A non est B*, idem est quod *QA est non B*", A VI, 4, 808, § 9], but when he later offers a proof of this principle in § 18, he uses the unambiguous and explicit formulation "there exists a Q such that QA is $\overline{B}$" ["*datur Q tale ut QA sit non B*"].

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11 Cf. GI, § 21: "Deinde definitas a me significari prioribus Alphabeti literis, indefinitas posterioribus, nisi aliud significetur."

12 Parkinson [1966: 56]; cf. also §§ 17, 158, 189 and 198 GI or C., 301. In the "Specimina Calculi Rationalis" Leibniz used the letter 'L' as an "indefinite concept": "*A est B*, sic exponitur literaliter *A* ∞ *LB*, ubi *L* idem quod indefinitum quoddam" (A VI, 4, 808); cf. also A VI, 4, 812: "cum A est B dici potest *A* ∞ *LB* [...] potest per *L* intelligi Ens vel aliud quiddam quod jam in *A* continetur".

13 The '*' is meant to indicate that the principle is not (entirely) correct.

Now, there is a minor problem connected with NEG 6*. In view of CONJ 2, the concept $\overline{B}A$ contains $\overline{B}$; hence, trivially, there always exists at least one Y such that $YA\varepsilon\overline{B}$, namely $Y = \overline{B}$. Therefore one should improve NEG 6* by saying more exactly that the negation of the U.A., ‚Some A is not B', is true if and only if for some Y which is *compatible with A*: YA contains $\overline{B}$:

(NEG 6) $\neg(A\varepsilon B) \leftrightarrow \exists Y(P(YA) \wedge YA\varepsilon\overline{B})$.

As a matter of fact, Leibniz himself hit upon the necessity of postulating the self-consistency of QA when he proved NEG 6 by means of the POSS 1 as follows:

"'A is not B' and 'QA is non B' coincide, i.e. to say 'A isn't B' is the same as to say 'there exists a Q such that QA is non B'. If 'A is B' is false, then '*A non B' is possible* by [POSS 1]. 'Non B' shall be called 'Q'. Therefore *QA is possible*."[14]

In other places, however, Leibniz often overlooked this requirement or he simply took the self-consistency of the corresponding concept for granted. Thus in §§ 47, 48 GI after stating that "'A contains B' is a universal affirmative in respect of A" he suggests the following formalization for the P.A.: "'AY contains B' is a particular affirmative in respect of A". Since $AY\varepsilon B$, i.e. more explicitly $\exists Y(AY\varepsilon B)$, follows from the trivial law $AB\varepsilon B$, this condition cannot adequately express the content of the PA which rather has to be formalized by $\exists Y(P(AY) \wedge AY\varepsilon B)$.

Let us now turn to the basic principles of a logic of quantifiers! The inference of so-called existential generalization,

(EXIS 1) $\phi(A) \vdash \exists Y\phi(Y)$,

according to which any proposition asserting that a certain concept A has the property ϕ entails that for some indefinite concept Y $\phi(Y)$ holds, was formulated in § 23 GI as follows: "For any definite letter there can be substituted an indefinite letter not yet used [...] i.e. one can put A=Y". Furthermore Leibniz provided several special instances or applications of this rule, e.g.:

(EXIS 1.1) $A=AA \vdash \exists Y(A=AY)$
(EXIS 1.2) $AB\in C \vdash \exists Y(AY\in C)$
(EXIS 1.3) $A=AB \vdash \exists Y(A=YB)$.

Thus in § 24 GI he derives $\exists Y(A=AY)$ from the principle of idempotence, CONJ 4, by noting:

"To any letter a new indefinite one can be added; e.g., for A we can put AY. For A = AA (by [CONJ 4]), and A is Y (or, for A one can put Y, by [EXIS 1]); therefore A = AY" (Parkinson [1966: 57]).

In § 49 GI he proves EXIS 1.2 as follows: "If AB is C, it follows that AY is C; or, it follows that some A is C. For it can be assumed by [EXIS 1] that B=Y" (Parkinson [1966: 59]).

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14 Cf. A VI, 4, 809: "*A non est B* et *QA est non B* coincidere seu dicere *A non est B*, idem esse ac dicere *datur Q, tale ut QA sit non B.* Si falsum est *A est B*, possibile est *A non B* per [POSS 1], *non B* vocetur *Q*, ergo possibile est *QA*".

Furthermore, the validity of EXIS 1.3 (that had already been maintained in § 117 GI)[15] was proved, e.g., in a fragment of August 1st, 1690 as follows: "If A = AB, there can be assumed a Y such that A = YB. This is a postulate but it can also be proved, for A itself at any rate can be designated by Y" (Parkinson [1966: 90]). In # 13 of the same fragment Leibniz also shows the converse implication: "If A = YB, it follows that A = AB. I prove this as follows. A = YB (by hypothesis), therefore AB = YBB (by [11][16]) = YB (by [CONJ 4]) = A (by hypothesis)."

These examples should suffice to show that Leibniz had a fairly good understanding of the rule for *introducing* an existential quantifier, EXIS 1. Moreover, one may also ascribe to him at least a partial insight into the validity of the converse rule for *eliminating* existential quantifiers. In modern systems of natural deduction this rule says that from an existential proposition of the form $\exists Y\alpha[Y]$ one may deduce a corresponding singular proposition $\alpha[A]$ provided that the singular term A is a "new" one, which does not yet occur in the corresponding context:

(EXIS 2) $\exists Y\phi(Y) \vdash \phi(A)$, for some "new" constant A.

In this vein Leibniz notes in GI § 27:

"Some B = YB, and therefore some A = ZA [...] *but a new indefinite letter, namely Z*, is to be assumed for the latter equation just as Y had been assumed a little earlier" (Parkinson [1966: 57]; my emphasis).

This passage may be interpreted as saying that from a proposition, e.g., of the form ‚Some A is C', i.e. $\exists Y(AY\varepsilon C)$, one may deduce that $AZ\varepsilon C$, provided that the indefinite concept Z is "new". In Lenzen [1984a] various other examples were discussed which show that Leibniz often applied the rule of inference, EXIS 2, is just this sense.

Although Leibniz did not always fully realize the difference between the use of indefinite concepts functioning as *existential* and as *universal* quantifiers, he sometimes at least partly recognized that the negation of a formula containing an indefinite concept as an *existential* quantifier gives rise to a *universally* quantified proposition. Thus in a somewhat confused passage of § 112 GI[17] he said:

"It must be seen whether, when it is said that AY is B (i.e. that some A is B), Y is not taken in some other sense than when it is denied that any A is B, in such a way that not only is it denied that some A is B – i.e. that this indeterminate A is B – but also that any A out of a number of indeterminates is B, so that when it is said that no A is B, the sense is that it is denied that $A\hat{Y}$ is B; for $\hat{Y}$ is Y, i.e. any Y will contain this Y. So when I say that some A is B, I say that this some ["hoc quoddam"] A is B; if I deny that some A is B, or that this some A is B, I seem only to state a particular negative. But when I deny that any A is B, i.e. that not only this, but also this and this A is B, then I deny that $\hat{Y}$ is B." (Parkinson [1966: 72]).

While the P.A. shall be formalized, according to Leibniz, by 'AYεB' with Y functioning as an *existential* quantifier, its negation shall not be represented as $\neg(AY\varepsilon B)$, but rather by means of a new symbol $\hat{Y}$ as $\neg(A\hat{Y}\varepsilon B)$, where this new type of indefinite concept $\hat{Y}$ denotes "any Y" ["quodcunque Y"] and thus represents a *universal* quantifier. To put it less elliptically:

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15 "A = BY is the same as that A = BA". Cf. also A VI, 4, 808: "Potest etiam sic exponi A ∞ AB, ut non sit opus assumi tertium".

16 The inference from A = YB to AB = YBB is licensed by principle # 11 of the same essay ("If A = C, AC = BC") and not, as the editions of Couturat and Parkinson have it, by # 10. It is true that the manuscript contains "per (10)", but this slip is due to the fact that Leibniz originally numbered the quoted principle as # (10), and when he later renumbered it as # 11, he forgot to change the reference accordingly.

17 In order to avoid confusion with our formalization of conceptual negation, the symbol $\overline{Y}$ which Leibniz here uses for the "universal" indeterminate concept was replaced by ‚$\hat{Y}$'. Cf. also §§ 80 – 82 GI where Leibniz similarly uses two different symbols for indefinite concepts.

whereas ‘Some A is B’ may be formalized in *L2* as $\exists Y(AY\varepsilon B)$[18], the negation takes the form $\forall \hat{Y}\neg(A\hat{Y}\varepsilon B)$ in accordance with the well-known law

(UNIV 1) $\neg\exists Y\alpha[Y] \leftrightarrow \forall Y\neg\alpha[Y]$.

In view of this explanation, Leibniz’s incidental remark “$\hat{Y}$ is Y, i.e. any Y will contain this Y” [“$\hat{Y}$ est Y, seu quodcunque Y continebit hoc Y”] expresses another important law of the logic of quantifiers, namely: Each proposition of the form $\alpha[\hat{Y}]$ entails the corresponding proposition $\alpha[Y]$, or less elliptically:

(UNIV 2) $\forall Y\alpha[Y] \rightarrow \exists Y\alpha[Y]$.

This principle was anticipated in the fragment “De Propositionibus Existentialibus” where Leibniz had similarly used two types of indefinite concepts, Y and $\tilde{Y}$:

“Let us see in which way Y and $\tilde{Y}$ differ from each other, namely like ‘something’ and ‘whatsoever’ but this happens by accident, and I want it to be Y simpliciter. This must be examined more carefully.”[19]

Unfortunately, Leibniz never carried out the closer examination of this topic. Nevertheless it should be clear that Y as ‘something’ represents the existential quantifier $\exists Y$ while $\tilde{Y}$ as ‘whatsoever’ corresponds to the universal quantifier $\forall Y$, and the remark that $\tilde{Y}$ “is Y simpliciter” may be taken to mean that a universal proposition of the type $\forall\tilde{Y}\alpha[\tilde{Y}]$ *entails* the corresponding existential proposition $\exists Y\alpha[Y]$.

There are various other logical laws where Leibniz used indefinite concepts as universal quantifiers. Thus in A VI, 4, 808 he formulates: “15) A is B is the same as to say: If L is A, it follows L is also B” [“*A est B*, idem est ac dicere *si L est A sequitur quod et L est B*”]. Couturat [1901: 347, fn. 2] thought that this principle would represent only a variant of the “principe du syllogisme”, i.e. the law of transitivity of the ε-relation. But this interpretation is incompatible with the fact that CONT 2 has the form $A\varepsilon B \wedge L\varepsilon A \rightarrow L\varepsilon B$, or, equivalently, $A\varepsilon B \rightarrow (L\varepsilon A \rightarrow L\varepsilon B)$, where the first implication must never be strengthened into a biconditional! Furthermore Leibniz’s explanation “L is to be understood as any term of which ‘L is A’ can be said” [“Intelligitur autem *L* quicunque terminus de quo dici potest *L est A*”] makes clear that here L is not a definite but an indefinite concept, i.e. a *variable* functioning as a universal quantifier. Therefore the principle has to be formalized more explicitly as follows:

(UNIV 3) $(A\varepsilon B) \leftrightarrow \forall L(L\varepsilon A \rightarrow L\varepsilon B)$.

Leibniz’s proof contains an anticipation of the contemporary rules for eliminating and introducing universal quantifiers:

“Let us assume the proposition ‘A is B’. I say that it entails ‘If L is A, it follows that L is B’, which I prove as follows: Since A is B, hence A = AB […]. But if L is A, then L = LA. Whereby (substituting for A the value AB) one obtains L = LAB. Therefore L is AB, hence L is B […]. Now let us conversely prove that ‘If L is A, it follows that L is B’ entails ‘A is B’. L however is to be understood as any term of which ‘L is A’ can be said. So

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18 More exactly, in view of the trivial law $AB\varepsilon B$, the P.A. should be formalized by $\exists Y(P(AY) \wedge AY\varepsilon B)$ - cf. the discussion of principle NEG 6* above; this complication can, however, be ignored here.

19 Cf. A VI, 4, 1631: „Videndum quomodo [Y] et [$\tilde{Y}$] differant, scilicet ut aliquod et quodcunque sed id contingit per accidens, et velim qui sit Y simpliciter. Haec melius examinanda“. Note that for typographical reasons ‚X’ has here been replaced by ‚Y’ because my word processor only generates ‚$\tilde{Y}$’ but not Leibniz’s sign composed of an ‚X’ and ‚~’.

assume the one [$\forall$L(LεA $\rightarrow$ LεB)] to be true and yet the other [AεB] to be false. […] Therefore the following proposition will be stated: QA is non-B. […] But QA is A. Therefore QA is B (*because QA is subsumed under L*). Hence QA is B non-B what is absurd."[20]

In the first part Leibniz derives [$\forall$L](LεA $\rightarrow$ LεB) from the premiss AεB by showing that, for any L, LεA (in conjunction with AεB) entails LεB. This follows the basic idea of the rule of $\forall$-introduction according to which $\forall$Yα[Y] may be established by showing that, for any arbitrary concept A, α[A]. In the second part Leibniz proves indirectly that $\neg$(AεB) is incompatible with the premise [$\forall$L](LεA $\rightarrow$ LεB), because if AεB were false, then according to NEG 6 there would exist some Q such that QA$\varepsilon\overline{\text{B}}$ (and P(QA)); now, trivially, according to CONJ 3 QAεA; thus [$\forall$L](LεA $\rightarrow$ LεB) would allow us to conclude that QAεB ("because QA is subsumed under [the variable] L"); hence by CONJ 1 we would obtain QAεB$\overline{\text{B}}$ which is "absurd" or, more correctly, which contradicts P(QA). This proof follows the basic idea of $\forall$-elimination according to which $\forall$Yα[Y] entails, for any arbitrary A, α[A]:

(UNIV 4) $\forall$Yα[Y] $\rightarrow$ α[A]

4 Leibniz's Logical Criteria for Individual Concepts

Within the framework of system *L2*, it now becomes possible to formalize Leibniz's own criteria for the completeness of individual concepts. Let us start with a closer examination of §§ 71, 72 GI where Leibniz presents his views on *existence* and on *individuals*:

"(71) What is to be said about the proposition 'A is an existent' or 'A exists'? Thus, if I say about an existing thing, 'A is B', it is the same as if I were to say 'AB is an existent'; e.g. 'Peter is a denier', i.e. 'Peter denying is an existent'. The question here is how one is to proceed in analysing this; i.e. whether the term 'Peter denying' involves existence, or whether 'Peter existent' involves denial - or whether 'Peter' involves both existence and denial, as if you were to say 'Peter is an actual denier', i.e. is an existent denier; which is certainly true. Undoubtedly, one must speak in this way; and this is the difference between an individual or complete term and another. For if I say 'Some man is a denier', 'man' does not contain 'denial', as it is an incomplete term, nor does 'man' contain all that can be said of that of which it can itself be said.

(72) So if we have BY, and the indefinite term Y is superfluous (i.e., in the way that 'a certain Alexander the Great' and 'Alexander the Great' are the same), then B is an individual. If there is a term BA and B is an individual, A will be superfluous; or if BA=C, then B=C." (Parkinson [1966: 65])

Leibniz has often been blamed for not carefully distinguishing between terms and their denotations.[21] The quoted passage certainly justifies such a criticism, but Leibniz's rather careless use of the word 'individual' to refer either to individual- concepts or to individuals should not give rise to serious misunderstandings. One may assume that there is a one-to-one-correspondence between individuals and individual-terms, and the context makes perfectly clear what Leibniz is talking about.

For reasons of space the main topic of § 71, i.e. the issue of *existence*, cannot further be discussed here. It would lead us far astray to explain what Leibniz may have had in mind

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20 Cf. A VI, 4, 808/9: „Assumamus hanc propositionem *A est B*, dico hinc inferri *si L est A, sequitur quod L est B*. Hoc ita demonstro: Quia *A est B*, ergo *A* ∞ *AB* […]. Jam si *L est A*, erit *L* ∞ *LA*. Ubi (pro *A* substituendo valorem *AB*) fit *L* ∞ *LAB*. Ergo *L est AB*. Ergo *L est B* […]. Nunc inverse demonstremus, ex hac: *Si L est A sequitur quod L est B*, vicisssim inferri *A est B*. Intelligitur autem *L* quicunque terminus de quo dici potest *L est A*. Ponamus illud [$\forall$L(LεA $\rightarrow$ LεB)] esse verum, et tamen hoc [A$\in$B] esse falsum. […] Statuatur ergo haec enuntiatio: *QA est non B*. […] Jam *QA est A*. Ergo *QA est B* (quia *QA* comprehenditur sub *L*), ergo *QA est B non B* quod est absurdum".

21 Cf., e.g., Mates [1986: 65, fn. 68].

when he answered the question "whether the term 'Peter denying' involves existence, or whether 'Peter existent' involves denial - or whether 'Peter' involves both existence and denial" by asserting "Undoubtedly, one must speak in this way; and *this is the difference between an individual or complete term and another*". The reader is referred to section 3.4 of Lenzen [1990] or to Lenzen [1991]. Let it just be noted that in the concluding sentence of this paragraph Leibniz vaguely puts forward the "Wild quantity thesis" when he says that from the truth of the particular proposition 'Some man is a denier' it does not follow that the universal proposition '[Every] man is denier' be true as well ["'man' does not contain 'denial'"]. Here one evidently has to add the unspoken claim that the corresponding inference from a particular to a universal proposition does hold if the subject term is an individual-concept.

Now let's turn to § 72 which contains Leibniz's most explicit and detailed discussion of the logical criteria for complete individual concepts! He calls a concept A "superfluous" (with respect to concept B) iff (for every C) BA=C entails that B=C. This condition may be simplified by requiring that A is already contained in B. For, on the one hand, substituting 'BA' for 'C' yields that BA=BA entails B=BA; conversely, if BA=B then (for any C) BA=C entails that B=C. Hence A is superfluous with respect to B just in case that B=BA, i.e. $B \in A$. Now, when Leibniz goes on to maintain "If there is a term BA and B is an individual, A will be superfluous; or, if BA = C, then B = C" (Parkinson [1966: 65, fn. 1]), he seems to maintain that any term A is superfluous with respect to any individual term B. But this is absurd since otherwise an individual-concept B would be "completely complete" in the sense of containing *every* concept A, in particular besides A also Non-A, and hence B would be inconsistent.

To resolve this difficulty, observe that Leibniz begins the sentence in question by saying "Si sit terminus BA" which Parkinson translated as "If there is a term BA". In other contexts, this translation would appropriately express the stipulation: "Let there be a term BA [...]". In the present context, however, Leibniz meant to say: "Let the term BA *be*", i.e. let BA *be a self-consistent term*, or, let us suppose that P(BA)! There are several passages within and without the GI where Leibniz paraphrases the condition of self-consistency of a concept A just by saying 'A is'. Therefore the interpretation of "Si sit terminus BA" as meaning 'Let BA be a possible term' is very plausible, and it entails the necessary condition: B is an individual-concept only if, unlike other concepts, B is complete in the sense of already containing any concept A with which it is *compatible*, i.e. for which P(BA) holds.

Since A stands for any arbitrary concept, it may be replaced by an indefinite concept Y and then be bound by a universal quantifier. If we furthermore abbreviate the condition that B is an individual(concept) by 'I(B)' we obtain the following formal condition: If I(B), then $\forall Y(P(BY) \rightarrow B\varepsilon Y)$. That this is actually what Leibniz had in mind is evidenced by the fact that conversely $\forall Y(P(BY) \rightarrow B\varepsilon Y)$ is recognized by him as a sufficient condition for B to be an individual-concept when he says: "So if BY is [possible], and the arbitrary indefinite term Y is superfluous, then B is an individual". Basically in accordance with Kauppi's interpretation, then, we obtain the following definition of individual-concepts:

(IND 3) $\quad I(B) \leftrightarrow P(B) \wedge \forall Y(P(BY) \rightarrow B\varepsilon Y).$

Here only the trivial condition P(B) not mentioned by Leibniz (and also not considered by Kauppi) has been added. In other words, an individual concept B is a self-consistent concept which contains every concept Y with which it is compatible.

Next consider Mates' criterion saying that an individual concept contains, for each concept Y, either Y or its negation. This can straightforwardly be formalized as follows:

(IND 4) $\quad I(B) \leftrightarrow \forall Y(B\varepsilon \overline{Y} \leftrightarrow \neg(B\varepsilon Y)).$

Both criteria are easily shown to be equivalent to one another. First, let us suppose that IND 4 holds and show that IND 3 must be satisfied. Since, according to IND 4, individual concepts contain, for any Y, either Y or its negation, but not both, it follows that B is self-consistent. Furthermore, let Y be any concept such that P(BY). Then, according to POSS 1, B does not contain $\overline{Y}$, so that IND 4 entails the desired conclusion BεY. Second, let us assume conversely that IND 3 holds and show how IND 4 can be derived. If, on the one hand, for some Y, $B\varepsilon\overline{Y}$, then B cannot also contain Y because otherwise B would be inconsistent in contradiction to premise P(B); if, on the other hand, B does not contain a certain concept Y, it follows by POSS 1 that $P(B\overline{Y})$; hence, according to IND 3, we obtain the desired conclusion $C\varepsilon\overline{Y}$.

It may, however, be doubted whether the logical criterion IND 4 was ever explicitly taken into account by Leibniz himself. Mates only hinted out to a passage from a series of definitions where Leibniz remarked that there exist as many individual substances as there are different "combinationes omnium attributorum compatibilium" (A VI, 4, 306). This observation certainly does establish a one-to-one correlation between an individual substance, c, and a corresponding individual concept, C, but it does not make entirely clear that such a "combination of all compatible attributes", C, satisfies the logical condition $\forall Y(C\varepsilon\overline{Y} \leftrightarrow \neg(C\varepsilon Y))$. Nevertheless Leibniz came very close to acknowledging IND 4 as a logical criterion for individual concepts when he pointed out in the "Calculi Universalis Investigationes" of 1679:

"[...] Termini *contradictorii* sunt, quorum unus est positivus alter negativus hujus positivi, ut *homo* et *non homo*. De his regula observanda est: si duae exhibeantur propositiones ejusdem *praecise subjecti* singularis quarum unius unus terminorum contradictoriorum, alterius alter sit praedicatum, tunc necessario unam propositionem esse veram et alteram falsam. Dico autem: *ejusdem praecise subjecti*, exempli causa hoc aurum est metallum, hoc aurum est non-metallum."[22]

The crucial issue here is that the "law" $B\varepsilon\overline{A} \leftrightarrow \neg(B\varepsilon A)$ holds only for the case where the subject B is an *individual* concept (as, e.g., 'Petrus Apostolus') but not for general concepts as, e.g., 'homo'. The text-critical apparatus in A VI, 4, 218 reveals that Leibniz was somewhat diffident about this decisive point. He began to illustrate the above quoted "regula" by the correct example "si dicam *Petrus Apostolus fuit Episcopus Romanus*, et *Petrus Apostolus non fuit Episcopus Romanus*" and then went on, erroneously, to generalize this law so as to hold also for general terms: "Aut si dicam *Omnis homo est doctus*. *Omnis homo est non doctus*". Then he noticed his error: "Imo hic patet me errasse, neque enim procedid regula." As has been shown elsewhere, Leibniz repeated this "comedy of errors" not only in the *General Inquiries* of 1986[23], but also in some late fragments of 1690[24]. Therefore criterion IND 4 can only be partially ascribed to Leibniz himself.

Next let us consider Mates' second criterion according to which an individual concept "cannot be the predicate term in a true proposition unless that proposition is reciprocal". Taken literally, this condition would have to be formalized as $\forall Y(Y\varepsilon B \rightarrow B\varepsilon Y)$. However, if Y is *inconsistent*, then according to POSS 4 Y contains each concept A, hence also each

22. A VI, 4, 217/8; a discussion of this important passage may be found in Lenzen [1986], esp. pp. 23-24.

23 Cf. GI § 82: „B non est A idem esse quod B est non-A"; § 92: „Non valet consequentia: Si A non est non-B, tunc A est B".

24 Cf., e.g., LH IV, 7 B 2, 1 where Leibniz originally wrote: "Si verum est A continere B, falsum est A continere non-B *et contra*" (my emphasis); he then detected his error and weakened the "law" into "Si verum est A continere B, falsum est A continere non B". Unfortunately, these preliminary versions are not contained in Couturat's edition of the fragment ("Principia Calculi Rationalis", cf. C. 230). The whole issue of 'B non est A' vs. 'B est non-A' is discussed at greater length in Lenzen [1986].

individual concept B. But, of course, in this case one does not want to require that B conversely contains Y. Therefore the implication $Y\varepsilon B \rightarrow B\varepsilon Y$ must be restricted to self-consistent concepts Y. This restriction alone, however, does not yet guarantee that also B itself is self-consistent – as a matter of fact, if B were inconsistent, then $B\varepsilon Y$ for every Y, and hence a fortiori $\forall Y(P(Y) \wedge Y\varepsilon B \rightarrow B\varepsilon Y)$. Thus Mates' condition must be improved as follows:

(IND 5) $\quad I(B) \leftrightarrow P(B) \wedge \forall Y(P(Y) \wedge Y\varepsilon B \rightarrow B\varepsilon Y)$.

Again one easily proves that this condition is equivalent to IND 3 (and thus also to IND 4). First, assuming IND 5, we immediately get IND 3 because, whenever P(BY) holds for some concept Y, then in view of the trivial law $BY\varepsilon C$ (CONJ 2) one can infer by IND 5 that $B\varepsilon BY$ from which the desired conclusion $B\varepsilon Y$ follows by CONJ 3 and by the transitivity of the ε-relation. If secondly we assume IND 3, we immediately obtain IND 5 since the premise $P(Y) \wedge Y\varepsilon B$ by CONT 3 gives us Y=YB or (by CONJ 5) Y = BY, hence P(Y) entails P(BY), from which the desired $B\varepsilon Y$ follows by IND 3.

In contrast to IND 4, the core idea of IND 5 (minus the consistency requirements P(B) and P(Y)) has clearly been put forward by Leibniz himself. Thus in the fragment mentioned by Mates Leibniz writes:

"Now, a singular substance is one which cannot be said of another one. Or, if a singular substance is said about some one, they will be the same. For if from the very fact that A is B one can infer that also B is A, i.e. that B and A are the same, then A or B shall be said to be a singular substance, or a thing subsisting by itself."[25]

Finally let us consider the "Wild quantity thesis" which says that in the case of a *singular* term B the proposition 'B is A' is equivalent both to the universal proposition 'Every B is A' and to the particular proposition 'Some B is A'. As regards the first part of this thesis, it is easily shown that $(B\varepsilon A) \leftrightarrow \forall Y(BY\varepsilon A)$ holds not only for individual concepts but for *any* concept B. On the one hand, if $B\varepsilon A$, then in view of $BY\varepsilon Y$ (CONJ 3) one gets $BY\varepsilon A$ for arbitrary Y; conversely, if $\forall Y(BY\varepsilon A)$, then choosing Y = B one obtains by UNIV 4 $BB\varepsilon A$, i.e. in view of CONJ 4 the desired $B\varepsilon A$.

The second "half" of the "Wild quantity thesis", however, gives us another logical criterion for individual concepts. It asserts that whenever I(B), then $B\varepsilon A$ is equivalent to the particular affirmative proposition 'Some B is A' which Leibniz usually renders as $YB\varepsilon A$, i.e. less elliptically as $\exists Y(YB\varepsilon A)$. Now, as was discussed in connection with principle NEG 6* above, this formalization must further be amended by adding the requirement that YB is self-consistent: $\exists Y(P(YB) \wedge YB\varepsilon A)$. Hence if the subject of the proposition $B\varepsilon A$ is an individual concept, then $B\varepsilon A$ is equivalent to the particular affirmative proposition $\exists Y(P(YB) \wedge YB\varepsilon A))$. By way of generalization we thus obtain the further criterion

(IND 6) $\quad I(B) \leftrightarrow \forall X(B\varepsilon X \leftrightarrow \exists Y(P(YB) \wedge YB\varepsilon X))$.

Again IND 6 may be proven to be equivalent to the other conditions IND 3 – IND 5. First, assume that IND 3 (or IND 4 or IND 5) holds and consider any concept X such that $B\varepsilon X$; then, since I(B), one has according to IND 5 in particular P(B). In view of the trivial law BB = B, one thus obtains P(BB) and $BB\varepsilon X$ from which $\exists Y(P(YB) \wedge YB\varepsilon X))$ follows by EXIS 1.

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25 Cf. A VI, 4, 554-555: „Porro Substantia Singularis est quae de alio dici non potest. Seu si substantia singularis de aliquo dicatur erunt idem. Nempe si ex eo tantum quod *A* est *B* colligi potest etiam *B* esse *A*, seu *B* et *A* esse idem, dicetur *A* vel *B* esse *substantiam singularem*, sive rem per se subsistentem".

Conversely, if $\exists Y(P(YB) \wedge YB\varepsilon X))$, we know by IND 3 that B = BY, hence one obtains the desired $B\varepsilon X$. Second, assuming IND 6, it remains to be shown that, e.g., IND 4 is satisfied. Assuming (i) $B\varepsilon \overline{Y}$, we can infer by IND 6 that $\exists Z(P(ZB) \wedge ZB\varepsilon \overline{Y}))$; from this it easily follows that $\neg(B\varepsilon Y)$ because otherwise also $ZB\varepsilon Y$ and hence $ZB\varepsilon Y\overline{Y}$ (by CONJ 1), i.e. $\neg P(ZB)$, in contradiction to our assumption P(ZB). Finally, assuming (ii) $\neg(B\varepsilon Y)$, we know by POSS 1 that $P(\overline{Y}B)$ from which in view of the trivial $\overline{Y}B\varepsilon\overline{Y}$ one gets a fortiori $\exists X(P(XB) \wedge XB\varepsilon\overline{Y}))$ so that IND 6 allows us to derive the desired conclusion $B\varepsilon\overline{Y}$.

From a systematic point of view, it would remain to be shown that these criteria are *semantically adequate*; i.e., one would have to verify that B is an individual concept according to IND 3 – IND 6 iff, for each extensional interpretation ϕ, there exists exactly one possible individual b (from the underlying universe of discourse) such that $\phi(B) = \{b\}$. The technically somewhat complicated proof has been presented in section 3.3 of Lenzen [1990] and shall therefore not be repeated here.

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